

Helicity Amplitude for $\bar{p}p \rightarrow \omega\pi^0, \omega \rightarrow \pi^0\gamma, v^2$

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Abstract

The analysis of $\bar{p} p$ reactions in flight has been started up to now from the J^{PC} intermediate states(Standard Method).The problem with the Standard Method is,that additional Clebsch-Gordan Coefficients describing the coupling of the $\bar{p} p$ system with the J^{PC} system are not taken correctly into account.The same is true for the kinematical factors(different for different beam-energies).They are important in the comparison of amplitudes at different \bar{p} energies.Here,the whole reaction chain is described starting from the antiproton- and proton- helicity states,avoiding the above mentioned complications.That is done for the $\bar{p}p \rightarrow \omega\pi^0, \omega \rightarrow \pi^0\gamma$ reaction,but can be easily expanded to more general cases.A comparison of the results with the Standard Method is performed showing that the Standard Method does not reproduce the correct sign of the amplitudes.Also the spin density matrix of the omega is discussed and the determination of its elements using different methods is outlined..

1 Definitions for $\bar{p}p \rightarrow \omega\pi^0, \omega \rightarrow \pi^0\gamma$

The definitions of the relevant quantities for the reaction are given in Fig.1.

1.1 Quantum numbers of particles

The quantum numbers of the particles relevant for the reaction under discussion are summarized in Table 1.

1.2 Quantum numbers of Sub-Systems

12-System:

$$S_{12} = 0, 1; L_{12} = 0, 1, 2, 3, \dots; P_{12} = (-1)^{L_{12}+1}$$

$$S_{12} = 0 : J_{12} = L_{12} = 0, 1, 2, \dots$$

$$S_{12} = 1 : J_{12} = L_{12} \pm 1, L_{12}, = 0, 1, 2 \dots$$

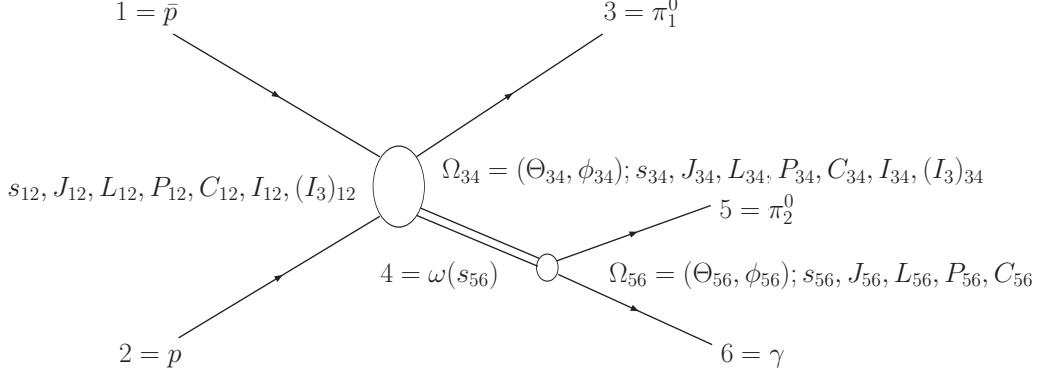


Figure 1: Definitions for the reaction $\bar{p}p \rightarrow \omega\pi^0, \omega \rightarrow \pi^0\gamma$. The helicity angles θ_{34}, Φ_{34} are measured in the overall CM-system with $\vec{z}\|\vec{p}_{\bar{p}}$. The helicity angles Θ_{56}, Φ_{56} are measured in the ω rest system, with $\vec{z}\|\vec{p}_{\omega}$.

Properties	$1 = \bar{p}$	$2 = p$	$3 = \pi_1^0$	$4 = \omega$	$5 = \pi_2^0$	$6 = \gamma$
Rest mass	$m_{\bar{p}}$	m_p	m_{π^0}	m_ω	m_{π^0}	$m_\gamma = 0$
Momentum	$\vec{p}_{\bar{p}}$	\vec{p}_p	$\vec{p}_{\pi_1^0}$	\vec{p}_ω	$\vec{p}_{\pi_2^0}$	\vec{p}_γ
Spin	$1/2$	$1/2$	0	1	0	1
Helicities	$\pm 1/2$	$\pm 1/2$	0	$\pm 1, 0$	0	± 1
Isospin I	$1/2$	$1/2$	1	0	1	$0, 1$
I_3	$-1/2$	$1/2$	0	0	0	$-$
Parity	-1	$+1$	-1	-1	-1	-1
C-Parity	$-$	$-$	$+1$	-1	$+1$	-1
G-Parity	$-$	$-$	-1	-1	-1	$-$

Table 1: Properties of the contributing particles

$$\begin{aligned} C_{12} &= (-1)^{L_{12}+S_{12}} \\ I_{12} &= 0, 1; (I_3)_{12} = 0 \\ G_{12} &= (-1)^{L_{12}+S_{12}+I_{12}} \end{aligned}$$

34-System:

$$\begin{aligned} S_{34} &= S_\omega = 1; L_{34} = 0, 1, 2, \dots; P_{34} = (-1)^{L_{34}} \\ J_{34} &= L_{34} \pm 1, L_{34}, = 0, 1, 2, \dots \\ C_{34} &= -1 \\ I_{34} &= 1 \\ G_{34} &= 1 \end{aligned}$$

56-System:

$$\begin{aligned} S_{56} &= 1; L_{56} = 1 (\text{Parity Conservation}); P_{56} = (-1)^{L_{56}} = P_\omega = -1 \\ J_{56} &= J_\omega = 1 \end{aligned}$$

$$I_{56} = 0, 1$$

1.3 Conserved Quantities

$$\begin{aligned} J_{12} &= J_{34} = J; J_\omega = 1 = J_{56}; P_{12} = P_{34} = P; C_{12} = C_{34} = C (= -1); P_{56} = P_\omega = -1 \\ I_{12} &= I_{34} = 1; (I_3)_{12} = (I_3)_{34} \quad (12 \rightarrow 34 \text{ transition:Strong interaction}) \\ (I_3)_{34} &= (I_3)_{56} \quad (34 \rightarrow 56 \text{ transition:Electromagnetic interaction}) \end{aligned}$$

1.4 CM-System

$$\begin{aligned} \vec{p}_p &= -\vec{p}_{\bar{p}}; |\vec{p}_{\bar{p}}| = |\vec{p}_p| = p_{\bar{p}} = p_{12} \\ \vec{p}_{\pi_1^0} &= -\vec{p}_\omega; |\vec{p}_{\pi_1^0}| = |\vec{p}_\omega| = p_\omega = p_{34} \\ s_{12} &= (\underline{p}_1 + \underline{p}_2)^2 = m_{\bar{p}}^2 + m_p^2 + 2E_{\bar{p}}E_p + 2p_{\bar{p}}^2 = (E_{\bar{p}} + E_p)^2 \\ (\underline{p} &\text{ means four-vector}) \\ s_{34} &= s_{12} \\ s_{56} &= m_\omega^2 \end{aligned}$$

2 Differential Cross Section

2.1 Differential cross section for unpolarized particles

For unpolarized antiprotons and protons the number of particles(d^5N) scattered into a phase space volume dLips is given by [1], [2]

$$d^5N = N_{initial} \times n_0 \times \Delta z \times d^5\sigma = L_{int} \times d^5\sigma \quad (n_0 = \rho_{Mol} \times A_{Av}) \quad (1)$$

with

$$\begin{aligned} d^5\sigma &= flux \times \overline{|T_{fi}|^2} \times PhaseSpace = \\ &= \frac{1}{4s^{1/2}p_{\bar{p}}} \left| \sum_{\lambda_4} a_{I1,I2,I3,I4}^{I3_1,I3_2,I3_3,I3_4} T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega) A_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi_2^0\gamma) \right|^2 \times \\ &\times \frac{dLips(s, p_3, p_5, p_6)}{(m_\omega^2 - s_\omega)^2 + m_\omega^2\Gamma_\omega^2} = \\ &= \frac{1}{4s^{1/2}p_{\bar{p}}} \times \frac{1}{2S_1 + 1} \frac{1}{2S_2 + 1} \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_5,\lambda_6} \left| \sum_{\lambda_4} a_{I1,I2,I3,I4}^{I3_1,I3_2,I3_3,I3_4} T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega) \times \right. \\ &\times \left. A_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi_2^0\gamma) \right|^2 \times \frac{dLips(s, p_3, p_5, p_6)}{(m_\omega^2 - s_\omega)^2 + m_\omega^2\Gamma_\omega^2} \quad (\lambda_3 = \lambda_5 = 0) \quad (2) \end{aligned}$$

and

$$a_{I1,I2,I3,I4}^{I3_1,I3_2,I3_3,I3_4} = \text{isospin dependent factor}$$

$$dLips(s, p_3, p_5, p_6) = \frac{ds_\omega}{2\pi} \times dLips(s, p_3, p_4) \times dLips(s_\omega, p_5, p_6) \quad (3)$$

$$dLips(s, p_3, p_4) = \frac{p_{34}}{16\pi^2\sqrt{s}} \times d\Omega_{34} = Kin_{34} \times d\Omega_{34} = Kin_{34} \times d\cos\Theta_{34} d\Phi_{34} \quad (4)$$

$$dLips(s_\omega, p_5, p_6) = \frac{p_{56}}{16\pi^2\sqrt{s_\omega}} \times d\Omega_{56} = Kin_{56} \times d\Omega_{56} = Kin_{56} \times d\cos\Theta_{56} d\Phi_{56} \quad (5)$$

$$\overline{|T|^2} = \frac{1}{2S_1+1} \frac{1}{2S_2+1} \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |T|^2 \quad (6)$$

The product $d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56}$ can be written in terms of the angle between production and decay plane ($\Phi_{56}' = \Phi_{56} - \Phi_{34}$)

$$d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56} = d\cos\Theta_{34} d\cos\Theta_{56} d\Phi_{34} d\Phi_{56}' \quad (7)$$

The amplitudes T_{fi} are defined according to the usual conventions [2, 3, 1, 4]

For the process $1+2 \rightarrow 3+4$: $\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \times \frac{p_{34}}{p_{12}} \times |T_{fi}|^2$
(Dimension of $T_{fi} = Dim[T_{fi}] = 1$)

For the decay $4 \rightarrow 5 + 6$: $\Gamma = \frac{1}{2m_4} \int |A_{fi}|^2 \times \frac{p_{56} d\Omega_{56}}{16\pi^2 m_4}$
(Dimension of $A_{fi} = Dim[A_{fi}] = \text{GeV}$)

Note: For polarized particles in the initial state ($1 + 2 \rightarrow 3 + 4$) $\overline{|T_{fi}|^2}$ has to be replaced by

$$\sum_{\lambda_1, \lambda_2, \lambda_1', \lambda_2'} \rho_{\lambda_1, \lambda_1'}^1 \rho_{\lambda_2, \lambda_2'}^2 \sum_{\lambda_3, \lambda_4} T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} \times T_{\lambda_1', \lambda_2', \lambda_3, \lambda_4}^*$$

with the spin density matrices ρ^1 , ρ^2 describing the polarization states of the initial particles.

2.2 Isospin Factor

$a_{I1,I2,I3,I4}^{I3_1,I3_2,I3_3,I3_4} = \langle I_3, I_3, I_4, I_3 | I_1, I_3, I_2, I_3 \rangle$ is the isospin-dependent part of the amplitude.

It can be expanded into isospin states of definite total isospin I^2 and total I3:

$$\begin{aligned} \langle I_3, I_{33}, I_4, I_{34} | I_1, I_{31}, I_2, I_{32} \rangle &= \\ &= \sum_{I_{12}, I_{312}, I_{34}, I_{334}} \langle I_3, I_{33}, I_4, I_{34} | I_{34}, I_{334} \rangle \langle I_{34}, I_{334} | I_{12}, I_{312} \rangle \langle I_{12}, I_{312} | I_1, I_{31}, I_2, I_{32} \rangle = \\ &= \sum_{I_{12}, I_{312}} \langle I_3, I_{33}, I_4, I_{34} | I_{12}, I_{312} \rangle \langle I_{12}, I_{312} | I_1, I_{31}, I_2, I_{32} \rangle \end{aligned} \quad (8)$$

with

$$\langle I_{34}, I_{334} | I_{12}, I_{312} \rangle = \delta_{I_{12} I_{34}} \delta_{I_{312} I_{334}} \quad (9)$$

Nomenclature for Clebsch-Gordan's: $\langle j_1, j_3, j_2, j_3 | J, J_3 \rangle$

Here: $I_{12} = 0, 1; I_{34} = 1; I_{312} = 0; I_{334} = 0$

$$\begin{aligned} a_{I_1, I_2, I_3, I_4}^{I_{31}, I_{32}, I_{33}, I_{34}} &= \langle I_3, I_{33}, I_4, I_{34} | I_1, I_{31}, I_2, I_{32} \rangle = \\ &= \langle 1/2, -1/2, 1/2, 1/2 | 00 \rangle \times \underbrace{\langle 0, 0 | 1, 0, 0, 0 \rangle}_0 + \\ &\quad + \langle 1/2, -1/2, 1/2, 1/2 | 1, 0 \rangle \times \underbrace{\langle 1, 0 | 1, 0, 0, 0 \rangle}_1 = 1/\sqrt{2} \end{aligned} \quad (10)$$

Here, particles 3 and 4 are C-Parity Eigenstates, so that $C_{34} = -1 = C_{12}$. There are, however, cases, e.g. $\bar{p}p \rightarrow K\bar{K}$, where states have to be constructed, which are eigenstates of I and C(G)[5].

2.3 Differential cross section for vanishing omega width

With

$$\lim_{m_\omega \Gamma_\omega \rightarrow 0} \frac{1}{(m_\omega^2 - s_\omega)^2 + m_\omega^2 \Gamma_\omega^2} = \frac{\pi}{m_\omega \Gamma_\omega} \delta(m_{omega}^2 - s_\omega)$$

(2) can be written, using (10)

$$\begin{aligned} d^5\sigma &= 1/2 \times \frac{1}{4s^{1/2} p_{\bar{p}}} \overline{\left| \sum_{\lambda_4} T_{\lambda_1, \lambda_2, \lambda_3 \lambda_4} (\bar{p}p \rightarrow \pi_1^0 \omega) A_{\lambda_4, \lambda_5 \lambda_6} (\omega \rightarrow \pi_2^0 \gamma) \right|^2} \times \\ &\quad \times \frac{\pi}{m_\omega \Gamma_\omega} \delta(m_{omega}^2 - s_\omega) \frac{ds_\omega}{2\pi} dLips(s, p_3, p_4) dLips(s_\omega, p_5, p_6) \end{aligned} \quad (11)$$

yielding

$$\begin{aligned}
d^4\sigma &= \int \frac{d^5\sigma}{ds_\omega} ds_\omega = 1/2 \times \frac{1}{4s^{1/2}p_{\bar{p}}} \frac{1}{2m_\omega\Gamma_\omega} \overline{\left| \sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega) A_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi_2^0\gamma) \right|^2} \times \\
&\quad \times dLips(s, p_3, p_4) dLips(s_\omega, p_5, p_6) \\
&= 1/4 \times 1/2 \times \frac{1}{4s^{1/2}p_{\bar{p}}} \frac{1}{2m_\omega\Gamma_\omega} \times \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_5,\lambda_6} \overline{\left| \sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega) A_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi_2^0\gamma) \right|^2} \times \\
&\quad \times dLips(s, p_3, p_4) dLips(s_\omega, p_5, p_6)
\end{aligned} \tag{12}$$

2.4 Expansion into partial waves

The $T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega)$ and $A_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi_2^0\gamma)$ amplitudes are expanded into partial waves [8] with $J=J_{12}=J_{34}$ and $J_{56}=J_\omega(=1)$

$$\begin{aligned}
&\left| \sum_{\lambda_4} T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega) A_{\lambda_4,\lambda_5\lambda_6}(\omega \rightarrow \pi_2^0\gamma) \right|^2 = \\
&= \left| \sum_{\lambda_4} \frac{8\pi s^{1/2}}{\sqrt{p_{\bar{p}}p_\omega}} \sum_J (2J+1) D_{\lambda_1-\lambda_2,\lambda_4}^{J*}(\Omega_{34}) \times \right. \\
&\quad \times \langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle \times \sqrt{\frac{2S_\omega + 1}{4\pi}} D_{\lambda_4\lambda_6-\lambda_5}^{1*}(\Omega_{56}) \times A_{\lambda_5,\lambda_6}^1 \Big|^2 = \\
&= \frac{48\pi s}{p_{\bar{p}}p_\omega} \left| \sum_J (2J+1) \sum_{\lambda_4} D_{\lambda_1-\lambda_2,\lambda_4}^{J*}(\Omega_{34}) D_{\lambda_4\lambda_6-\lambda_5}^{1*}(\Omega_{56}) \times \right. \\
&\quad \times \langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle \times A_{\lambda_5,\lambda_6}^1 \Big|^2 \quad (J = J_{12} = J_{34}; \lambda_3 = 0, \lambda_5 = 0)
\end{aligned} \tag{13}$$

with

$$T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega) = \sum_J \frac{8\pi s^{1/2}}{\sqrt{p_{\bar{p}}p_\omega}} (2J+1) D_{\lambda_1-\lambda_2,\lambda_4}^{J*}(\Omega_{34}) \times \langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle \tag{14}$$

$$(\text{Dim}[T_{\lambda_1,\lambda_2,\lambda_3\lambda_4}(\bar{p}p \rightarrow \pi_1^0\omega)] = \text{Dim}[\langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle] = 1)$$

and

$$A_{\lambda_4,\lambda_5\lambda_6}^{J=1}(\omega \rightarrow \pi_2^0\gamma) = \sqrt{\frac{2S_\omega + 1}{4\pi}} D_{\lambda_4\lambda_6-\lambda_5}^{1*}(\Omega_{56}) A_{\lambda_5,\lambda_6}^1 \tag{15}$$

$$(\text{Dim}[A_{\lambda_4, \lambda_5 \lambda_6}^{J=1}(\omega \rightarrow \pi_2^0 \gamma)] = \text{Dim}[A_{\lambda_5, \lambda_6}^1] = \text{GeV})$$

The definition of the partial wave amplitudes corresponds to the usual conventions [6, 7, 8]:

For the reaction $1 + 2 \rightarrow 3 + 4$:

$$\frac{d\sigma}{d\Omega_{34}} = \frac{1}{p_{12}^2} \left| \sum_J (2J+1) D_{\lambda_2 - \lambda_1, \lambda_4 - \lambda_3}^{J*} \langle \lambda_3, \lambda_4 | T^J | \lambda_1, \lambda_2 \rangle \right|^2$$

For the decay $4 \rightarrow 5+6$:

$$\Gamma = \frac{1}{2m_4} \frac{p_{56}}{16\pi^2 m_4} \times \int |A_{\lambda_4, \lambda_5 \lambda_6}^{J=1}(\omega \rightarrow \pi_2^0 \gamma)|^2 d\Omega_{56} = \frac{p_{56}}{32\pi^2 m_4^2} |A_{\lambda_5, \lambda_6}^1|^2$$

using $\int |D^J|^2 \times d\Omega_{56} = \frac{4\pi}{2J+1}$

2.5 Expansion in LS-Basis

The partial wave amplitudes $A_{\lambda_5, \lambda_6}^1$ and $\langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle$ are expanded in their LS-basis:

$\omega \rightarrow \pi^0 \gamma$ ($L_{56} = 1; S_{56} = 1$; only one term because of parity conservation):

$$\begin{aligned} A_{\lambda_5, \lambda_6}^1 &= \sqrt{\frac{2L_{56}+1}{2S_\omega+1}} \underbrace{\langle 1, 0, 1, \lambda_6 | 1, \lambda_6 \rangle}_{-\lambda_6 \sqrt{1/2}} \underbrace{\langle 0, 0, 1, \lambda_6 | 1, \lambda_6 \rangle}_1 \times \alpha_{L_{56}=1, S_{56}=1}^1 \\ &= -\frac{1}{\sqrt{2}} \times \lambda_6 \times \alpha_{11}^1 \quad (\alpha_{11}^1 = \text{const.}) \\ A_{0\lambda_6}^1 &= -A_{0-\lambda_6}^1 \quad (\text{Parity Conservation}) \end{aligned} \tag{16}$$

$$(\text{Dim}[\alpha_{11}^1] = \text{GeV}]$$

$\bar{p}p \rightarrow \omega \pi^0$:

$$\begin{aligned}
\langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle &= \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \underbrace{\langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle}_1 \times \\
&\times \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, \lambda_1 - \lambda_2 | J, \lambda_1 - \lambda_2 \rangle \langle 1/2, \lambda_1, 1/2, \lambda_2 | S_{12}, \lambda_1 - \lambda_2 \rangle \times \\
&\times \langle {}^{2L_{34}+1}L_{34_J} | T^J | {}^{2L_{12}+1}L_{12_J} \rangle \\
&= \sum_{L_{12}, S_{12}, L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{12}+1}{2J+1}} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \langle L_{12}, 0, S_{12}, \lambda_1 - \lambda_2 | J, \lambda_1 - \lambda_2 \rangle \times \\
&\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | S_{12}, \lambda_1 - \lambda_2 \rangle \times \langle {}^{2L_{34}+1}L_{34_J} | T^J | {}^{2L_{12}+1}L_{12_J} \rangle \quad (17) \\
&(\text{Dim}[\langle {}^{2L_{34}+1}L_{34_J} | T^J | {}^{2L_{12}+1}L_{12_J} \rangle] = 1)
\end{aligned}$$

For a fixed J -value, there exist the following L_{12}, S_{12} combinations:

For $J=0$:

$$\begin{aligned}
S_{12} = 0 : L_{12} &= 0 \\
S_{12} = 1 : L_{12} &= 1
\end{aligned}$$

For $J \geq 1$:

$$\begin{aligned}
S_{12} = 0 : L_{12} &= J \\
S_{12} = 1 : L_{12} &= J-1, J, J+1
\end{aligned}$$

In our case, for a given J there exist twelve ($3 \times 2 \times 2$) independent $\langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle$ -amplitudes. Their number is reduced to four (two), using Parity-and C-Parity-conservation.

Example: J=1

$$\begin{aligned}
\langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle = & \\
& = [\sqrt{1/3} \langle 0, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \underbrace{\langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle}_{1} \times \langle ^3 s_1 \\
& + \sqrt{2/3} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \underbrace{\langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle}_{1} \times \langle ^3 p_1 \\
& + \sqrt{5/3} \langle 2, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \underbrace{\langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle}_{1} \langle ^3 d_1] | T^1 | \times \\
& \times [\sqrt{3/3} \langle 1, 0, 0, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 0, \lambda_1 - \lambda_2 \rangle | ^1 P_1 \rangle + \\
& + \sqrt{1/3} \langle 0, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle | ^3 S_1 \rangle + \\
& + \sqrt{3/3} \langle 1, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle | ^3 P_1 \rangle + \\
& + \sqrt{5/3} \langle 2, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle | ^3 D_1 \rangle]
\end{aligned} \tag{18}$$

with $s, p, d, \dots \hat{=} L_{34} = 0, 1, 2, \dots ; S, P, D, \dots \hat{=} L_{12} = 0, 1, 2, \dots$

Because of Parity-and C-Parity conservation eight of the twelve amplitudes are zero:

Parity conservation: $P_{12} = (-1)^{L_{12}+1} = P_{34} = (-1)^{L_{34}}$

C-Parity conservation: $C_{12} = (-1)^{L_{12}+S_{12}} = C_{34} = -1$

The four remaining amplitudes are:

$$\begin{aligned}
\langle \lambda_4, \mathbf{0} | \mathbf{T}^1 | \lambda_1, \lambda_2 \rangle = & \\
& \sqrt{1/3} \langle 0, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \sqrt{3/3} \langle 1, 0, 0, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
& \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 0, \lambda_1 - \lambda_2 \rangle \langle ^3 s_1 | \mathbf{T}^1 | ^1 \mathbf{P}_1 \rangle + \\
& + \sqrt{2/3} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \sqrt{1/3} \langle 0, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
& \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \langle ^3 p_1 | \mathbf{T}^1 | ^3 \mathbf{S}_1 \rangle + \\
& + \sqrt{2/3} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \sqrt{5/3} \langle 2, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
& \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \langle ^3 p_1 | \mathbf{T}^1 | ^3 \mathbf{D}_1 \rangle + \\
& + \sqrt{5/3} \langle 2, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \sqrt{3/3} \langle 1, 0, 0, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
& \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 0, \lambda_1 - \lambda_2 \rangle \langle ^3 d_1 | \mathbf{T}^1 | ^1 \mathbf{P}_1 \rangle
\end{aligned} \tag{19}$$

For even J only two partial waves remain(see also [9])

Reason: Because of $(-1)^{L_{12}+S_{12}} = C_{34} = -1$, $L_{12} + S_{12}$ must be odd.

$S_{12} = 0 : L_{12} = J_{12} = \text{odd} \longrightarrow$ Term contributes only for $J_{12} = \text{odd}$

$S_{12} = 1 : L_{12} = J_{12} - 1, J_{12}, J_{12} + 1 = \text{even} \longrightarrow$ Term contributes once for $J_{12} = \text{even}$, two times for $J_{12} = \text{odd}$

A special case is J=0. Here, only 3P_0 and 1S_0 can contribute. Both have C=+1, so that in our case (C=-1) the J=0-amplitudes are zero.

Note: An alternative way to take the parity conservation into account [4] is the use of

$$\langle \lambda_4, 0 | T^J | \lambda_1, \lambda_2 \rangle = \eta_{\bar{p}} \times \eta_p \times \eta_{\omega} \times \eta_{\pi^0} \times (-1)^{S_{\omega} + S_{\pi^0} + S_{\bar{p}} + S_p} \times \langle -\lambda_4, 0 | T^J | -\lambda_1, -\lambda_2 \rangle \quad (20)$$

Trick in computing: Set always $\lambda_{\bar{p}} \equiv 1/2$

For taking into account C-conservation there is no direct way in the helicity formalism. An expansion in partial waves (like above) is mandatory.

2.6 Final Amplitude

From (12,13,16,17) the final expression for the cross section is derived:

$$\begin{aligned} \frac{d^4\sigma}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{34}d\Phi_{56'}} &= \frac{3p_{\gamma}}{4096\pi^3 p_p^2 m_{\omega}^2 \Gamma_{\omega}} \times \\ &\times \sum_{\lambda_1, \lambda_2, \lambda_6} \left| \sum_J (2J+1) \sum_{\lambda_4} D_{\lambda_1-\lambda_2, \lambda_4}^{J*}(\Omega_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \right. \\ &\times \sum_{L_{12}, S_{12}, L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{12}+1}{2J+1}} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \langle L_{12}, 0, S, \lambda_1 - \lambda_2 | J, \lambda_1 - \lambda_2 \rangle \times \\ &\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | S, \lambda_1 - \lambda_2 \rangle \times \lambda_6 \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{J^{PC}} \Big|^2 \end{aligned} \quad (21)$$

with

$$\Phi_{56'} = \Phi_{56} - \Phi_{34} \quad \text{and}$$

the Fit Parameters

$$\hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{J^{PC}} = \langle ^{2S_{34}+1}L_{34J} | T^J | ^{2S_{12}+1}L_{12J} \rangle \times \alpha_{11}^1$$

$$(\text{Dim}[\hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{J^{PC}}]) = \text{GeV}$$

$\hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{J^{PC}} \neq 0$ only for
 $P_{12} = (-1)^{L_{12}+1} = P_{34} = (-1)^{L_{34}}$ (Parity conservation) and
 $C_{12} = (-1)^{L_{12}+S_{12}} = C_{34} = -1$ (C-Parity conservation)

The number of fit parameters(4 complex numbers for J odd and two complex numbers for J even)increases with s (see [9, 10, 11])

For J=1 the expression reads

$$\begin{aligned}
 \frac{d^4\sigma}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{34}d\Phi_{56'}} &= \frac{3p_\gamma}{4096\pi^3 p_{\bar{p}}^2 m_\omega^2 \Gamma_\omega} \times \\
 &\times \sum_{\lambda_1, \lambda_2, \lambda_6} \left| \dots \dots (2 \times 1 + 1) \sum_{\lambda_4} D_{\lambda_1 - \lambda_2, \lambda_4}^{J*}(\Omega_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \right. \\
 &\times [\sqrt{1/3} \times \langle 0, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \times \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \times \sqrt{3/3} \langle 1, 0, 0, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
 &\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 0, \lambda_1 - \lambda_2 \rangle \times \hat{\mathbf{T}}_{1,0,0,1}^{1+-} + \\
 &+ \sqrt{2/3} \times \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \times \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \times \sqrt{1/3} \langle 0, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
 &\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \hat{\mathbf{T}}_{0,1,1,1}^{1--} + \\
 &+ \sqrt{2/3} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \times \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \sqrt{5/3} \langle 2, 0, 1, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
 &\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \hat{\mathbf{T}}_{2,1,1,1}^{1--} + \\
 &+ \sqrt{5/3} \langle 2, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \langle 1, \lambda_4, 0, 0 | 1, \lambda_4 \rangle \times [\sqrt{3/3} \langle 1, 0, 0, \lambda_1 - \lambda_2 | 1, \lambda_1 - \lambda_2 \rangle \times \\
 &\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | 0, \lambda_1 - \lambda_2 \rangle \times \hat{\mathbf{T}}_{1,0,2,1}^{1+-} \dots \dots] \times \lambda_6 \left. \right|^2 \quad (22)
 \end{aligned}$$

Integration over Φ_{34} with $\int |e^{iM\Phi_{34}}|^2 d\Phi_{34} = 2\pi$, for all M :

$$\begin{aligned}
 \frac{d^3\sigma}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{56'}} &= \int \frac{d^4\sigma}{d\cos\Theta_{34}d\cos\Theta_{56}d\Phi_{34}d\Phi_{56'}} d\Phi_{34} = \\
 &= 2\pi \times \frac{3p_\gamma}{4096\pi^3 p_{\bar{p}}^2 m_\omega^2 \Gamma_\omega} \times \sum_{\lambda_1, \lambda_2, \lambda_6} \left| \sum_J (2J + 1) \sum_{\lambda_4} d_{\lambda_1 - \lambda_2, \lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \right. \\
 &\times \sum_{L_{12}, S_{12}, L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{12} + 1}{2J + 1}} \sqrt{\frac{2L_{34} + 1}{2J + 1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \langle L_{12}, 0, S_{12}, \lambda_1 - \lambda_2 | J, \lambda_1 - \lambda_2 \rangle \times \\
 &\times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | S_{12}, \lambda_1 - \lambda_2 \rangle \times \lambda_6 \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{JPC} \left. \right|^2 = \\
 &= 2\pi \times \frac{3p_\gamma}{4096\pi^3 p_{\bar{p}}^2 m_\omega^2 \Gamma_\omega} \times w \quad (23)
 \end{aligned}$$

with w = weight of the event.

That means:When you compare runs at different \bar{p} -momenta,you have to normalize by the different N_{init} values and by $\frac{1}{p_{\bar{p}}^2}$.

In the following the decomposition into pairs of λ_1 and λ_2 is performed. λ_6 is set to +1. The term with $\lambda_6 = -1$ is similar to the one discussed here and exhibits the same amplitudes.

$$\begin{aligned}
w = & \left| \sum_J (2J+1) \sum_{\lambda_4} d_{0,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, 0 | J, 0 \rangle \times \right. \\
& \times \langle 1/2, 1/2, 1/2, -1/2 | S, 0 \rangle \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}=1}^{J^{PC}} \Big|^2 + \quad (\lambda_1 = 1/2; \lambda_2 = 1/2; M = \lambda_1 - \lambda_2 = 0) \\
& + \left| \sum_J (2J+1) \sum_{\lambda_4} d_{0,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, 0 | J, 0 \rangle \times \right. \\
& \langle 1/2, -1/2, 1/2, 1/2 | S, 0 \rangle \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}=1}^{J^{PC}} \Big|^2 + \quad (\lambda_1 = -1/2; \lambda_2 = -1/2; M = 0) \\
& + \left| \sum_J (2J+1) \sum_{\lambda_4} d_{1,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, 1 | J, 1 \rangle \times \right. \\
& \times \langle 1/2, 1/2, 1/2, 1/2 | S_{12}, 1 \rangle \times \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}=1}^{J^{PC}} \Big|^2 + \quad (\lambda_1 = 1/2; \lambda_2 = -1/2; M = 1) \\
& + \left| \sum_J (2J+1) \sum_{\lambda_4} d_{-1,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, -1 | J, -1 \rangle \times \right. \\
& \times \langle 1/2, -1/2, 1/2, -1/2 | S_{12}, -1 \rangle \times \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2l+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}=1}^{J^{PC}} \Big|^2 \quad (\lambda_1 = -1/2; \lambda_2 = 1/2; M = -1) \tag{24}
\end{aligned}$$

The two terms with $M=0$ contain singlett ($S_{12}=0$) and triplett ($S_{12}=1$) contributions. The terms with $M=1$ and $M=-1$ contain only triplett contributions. The two $M=0$ -terms are ordered according to these contributions:

$$\begin{aligned}
w = & \left| \sum_J \left[\cdots \sum_{L_{12}, S_{12}(=0)} \cdots \langle L_{12}, 0, 0, 0 | J, 0 \rangle \times \underbrace{\langle 1/2, 1/2, 1/2, -1/2 | 0, 0 \rangle}_{\sqrt{1/2}} \right. \right. \times \\
& \times \sum_{L_{34}, S_{34}(=1)} \cdots \hat{T}_{L_{12}, 0, L_{34}, S_{34}(=1)}^{J^{PC}}(singlett) + \\
& + \cdots \sum_{L_{12}, S_{12}(=1),} \cdots \langle L_{12}, 0, 1, 0 | J, 0 \rangle \times \underbrace{\langle 1/2, 1/2, 1/2, -1/2 | 1, 0 \rangle}_{\sqrt{1/2}} \times \\
& \times \left. \sum_{L_{34}, S_{34}(=1)} \cdots \hat{T}_{L_{12}, 1, L_{34}, S_{34}(=1)}^{J^{PC}}(triplett)] \right|^2 + \\
& + \left| \sum_J \left[\cdots \sum_{L_{12}, S_{12}(=0)} \cdots \langle L_{12}, 0, 0, 0 | J, 0 \rangle \times \underbrace{\langle 1/2, -1/2, 1/2, 1/2 | 0, 0 \rangle}_{-\sqrt{1/2}} \right. \right. \times \\
& \times \sum_{L_{34}, S_{34}(=1)} \cdots \hat{T}_{L_{12}, 0, L_{34}, S_{34}(=1)}^{J^{PC}}(singlett) + \\
& + \cdots \sum_{L_{12}, S_{12}(=1),} \cdots \langle L_{12}, 0, 1, 0 | J, 0 \rangle \times \underbrace{\langle 1/2, 1/2, 1/2, -1/2 | 1, 0 \rangle}_{\sqrt{1/2}} \times \\
& \times \left. \sum_{L_{34}, S_{34}(=1)} \cdots \hat{T}_{L_{12}, 1, L_{34}, S_{34}(=1)}^{J^{PC}}(triplett)] \right|^2 + \\
& + \left| \cdots \text{as above} \cdots \right|^2 + \quad (M = 1) \\
& + \left| \cdots \text{as above} \cdots \right|^2 + \quad (M = -1)
\end{aligned} \tag{25}$$

The M=0 terms have the following structure(A,B complex numbers):

$$|A + B|^2 + |-A + B|^2 = 2 \times |A|^2 + 2 \times |B|^2 \tag{26}$$

With that identity one obtains the final expression for the amplitude:

$$\begin{aligned}
w = 2 \times & \left| \sum_J (2J+1) \sum_{\lambda_4} d_{0,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sum_{L_{12}, S_{12}=0,} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 0, 0 \mid J, 0 \rangle \times \right. \\
& \times \underbrace{\langle 1/2, 1/2, 1/2, -1/2 \mid 0, 0 \rangle}_{\sqrt{1/2}} \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, 0, L_{34}, S_{34}=1}^{J^{PC}}(singlett) \Big|^2 + \quad (M = 0, singlett) \\
& + 2 \times \left| \sum_J (2J+1) \sum_{\lambda_4} d_{0,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, S_{12}=1,} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 1, 0 \mid J, 0 \rangle \times \right. \\
& \underbrace{\langle 1/2, -1/2, 1/2, 1/2 \mid 1, 0 \rangle}_{\sqrt{1/2}} \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, 1, L_{34}, S_{34}=1}^{J^{PC}}(triplett) \Big|^2 + \quad (M = 0, tripbett) \\
& + \left| \sum_J (2J+1) \sum_{\lambda_4} d_{1,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, S_{12}=1,} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 1, 1 \mid J, 1 \rangle \times \right. \\
& \times \underbrace{\langle 1/2, 1/2, 1/2, 1/2 \mid 1, 1 \rangle}_1 \times \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, 1, L_{34}, S_{34}=1}^{J^{PC}}(triplett) \Big|^2 + \quad (M = 1, triplett) \\
& + \left| \sum_J (2J+1) \sum_{\lambda_4} d_{-1,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, 1} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, 1, -1 \mid J, -1 \rangle \times \right. \\
& \underbrace{\langle 1/2, -1/2, 1/2, -1/2 \mid 1, -1 \rangle}_1 \times \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 \mid J, \lambda_4 \rangle \times \\
& \times \hat{T}_{L_{12}, 1, L_{34}, S_{34}=1}^{J^{PC}}(triplett) \Big|^2 \quad (M = -1, triplett) \tag{27}
\end{aligned}$$

Note: Only terms with L_{12} , S_{12} , L_{34} and S_{34} compatible with the conservation of J,P and C are $\neq 0$.

Together with (23) this is the final expression for the calculation of the cross section(weight).

Examples are given for J=1 and J=2:

J=1:

$$\begin{aligned} S_{12}=0 & \quad L_{12}=\text{J=1(C-conservation)} \quad L_{34}=0, \mathcal{A}, 2(\text{P-conservation}) \\ S_{12}=1 & \quad L_{12} = 0, \mathcal{A}, 2(\text{C-conservation}) \quad L_{34} = \emptyset, 1, \mathcal{B}(\text{P-conservation}) \end{aligned}$$

$$\begin{aligned} w = 2 \times & \left| 3 \sum_{\lambda_4} d_{0,\lambda_4}^1(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{1/2} [\sqrt{1/3} \langle 0, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \times T_{1001}^{1+-} + \right. \\ & \left. + \sqrt{5/3} \langle 2, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \times T_{1021}^{1+-}] \right|^2 + \\ & + 2 \times \left| 3 \sum_{\lambda_4} d_{0,\lambda_4}^1(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{1/2} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle [\sqrt{1/3} \times T_{0111}^{1--} - \sqrt{5/3} \times T_{2111}^{1--}] \right|^2 + \\ & + \left| 3 \sum_{\lambda_4} d_{1,\lambda_4}^1(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{1/2} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle [\sqrt{1/3} \times T_{0111}^{1--} + \sqrt{1/6} \times T_{2111}^{1--}] \right|^2 + \\ & + \left| 3 \sum_{\lambda_4} d_{-1,\lambda_4}^1(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{1/2} \langle 1, 0, 1, \lambda_4 | 1, \lambda_4 \rangle [\sqrt{1/3} \times T_{0111}^{1--} + \sqrt{1/6} \times T_{2111}^{1--}] \right|^2 \quad (28) \end{aligned}$$

with the four independent amplitudes:

$$\begin{aligned} T_{1001}^{1+-} &= \langle ^3S_1 | \hat{T} | ^1P_1 \rangle \\ T_{1021}^{1+-} &= \langle ^3D_1 | \hat{T} | ^1P_1 \rangle \\ T_{0111}^{1--} &= \langle ^3P_1 | \hat{T} | ^3S_1 \rangle \\ T_{2111}^{1--} &= \langle ^3P_1 | \hat{T} | ^3D_1 \rangle \end{aligned}$$

J=2:

$$\begin{aligned} S_{12}=0 & \quad L_{12} = J = \mathcal{B}(\text{C-conservation}) \\ S_{12}=1 & \quad L_{12} = \mathcal{A}, 2 \quad \mathcal{B}(\text{C-conservation}) \quad L_{34} = 1, \mathcal{B}, 3(\text{P-conservation}) \end{aligned}$$

$$\begin{aligned} w = 2 \times & \left| 3 \sum_{\lambda_4} d_{0,\lambda_4}^2(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{1/2} [\sqrt{1/3} \langle 0, 0, 1, \lambda_4 | 1, \lambda_4 \rangle \times 0 \right. \\ & + 2 \times \left| 5 \sum_{\lambda_4} d_{0,\lambda_4}^2(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{5/3} \underbrace{\langle 2, 0, 1, 0 | 2, 0 \rangle}_0 \sqrt{1/2} [\sqrt{3/3} \langle 1, 0, 1, \lambda_4 | 2, \lambda_4 \rangle \times T_{2111}^{2-+} + \right. \\ & + \sqrt{7/3} \langle 3, 0, 1, \lambda_4 | 2, \lambda_4 \rangle \times T_{2131}^{1-+}] \right|^2 + \\ & + \left| 5 \sum_{\lambda_4} d_{1,\lambda_4}^2(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{5/3} \underbrace{\langle 2, 0, 1, 1 | 2, 1 \rangle}_{-\sqrt{1/2}} [\sqrt{3/3} \langle 1, 0, 1, \lambda_4 | 2, \lambda_4 \rangle \times T_{2111}^{2-+} + \right. \\ & + , \sqrt{7/3} \langle 3, 0, 1, \lambda_4 | 2, \lambda_4 \rangle \times T_{2131}^{1-+}] \right|^2 + \\ & + \left| 5 \sum_{\lambda_4} d_{-1,\lambda_4}^2(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \sqrt{5/3} \underbrace{\langle 2, 0, 1, -1 | 2, -1 \rangle}_{-\sqrt{1/2}} [\sqrt{3/3} \langle 1, 0, 1, \lambda_4 | 2, \lambda_4 \rangle \times T_{2111}^{2-+} + \right. \\ & + \sqrt{7/3} \langle 3, 0, 1, \lambda_4 | 2, \lambda_4 \rangle \times T_{2131}^{1-+}] \right|^2 \quad (29) \end{aligned}$$

with the two independent amplitudes

$$T_{2111}^{2-+} = \langle ^3P_2 | \hat{T} | ^3D_2 \rangle$$

$$T_{2111}^{2-+} = \langle {}^3f_2 | \hat{T} | 3D_2 \rangle$$

Note: The M=1 amplitude contributes in \sum_J with a negative sign, the M=-1 amplitude with a positive sign.

2.7 Comparison with the Standard Method

The Standard Method was used in various analyses of $\bar{p}p$ reactions in flight and is described in detail in [10],[11] and [9], e.g. It is based on the decays of states with a definite J^{PC} value, but the coupling to the $\bar{p}p$ helicity states is only taken crudely into account.

The contributing J^{PC} states up to $J=6$ are given in Table 2 (from [10]).

J	Singlett $\lambda = 0$	J^{PC}	Triplet $\lambda = \pm 1$	J^{PC}	Triplet $\lambda = 0, \pm 1$
0	1S_0	0^{-+}			3P_0 0^{++}
1	1P_1	1^{+-}	3P_1 1^{++}		${}^3S_1, {}^3D_1$ 1^{--}
2	1D_2	2^{-+}	3D_2 2^{--}		${}^3P_2, {}^3F_2$ 2^{++}
3	1F_3	3^{+-}	3F_3 3^{++}		${}^3D_3, {}^3G_3$ 3^{--}
4	1G_4	4^{-+}	3G_4 4^{--}		${}^3F_4, {}^3H_4$ 4^{++}
5	1H_5	5^{+-}	3H_5 5^{++}		${}^3G_5, {}^3I_5$ 5^{--}
6	1I_6	6^{-+}	3I_6 6^{--}		${}^3H_6, {}^3J_6$ 6^{++}

Table 2: Fermion-Antifermion initial states

The weight factor is given by [10]

$$w = 2|Singl., M = 0|^2 + 2|Tripl., M = 0|^2 + |Tripl., M = 1|^2 + |Tripl., M = -1|^2$$

The helicity description reproduces the contributing initial J^{PC} states. Non contributing terms are zero either because of C- or P-violation or zero C.G. coefficients due to the coupling of the J^{PC} states to the initial $\bar{p}p$ states.

Also the structure of the weight factor is reproduced, as well as the number of fit parameters per given J-value.

The amplitude of the Standard Method for the $\bar{p}p \rightarrow \omega + \pi^0; \omega \rightarrow \pi^0 + \gamma$ reaction for a given J,M and λ_6 is given by [9]

$$A_J^{M,\lambda_6} = \sum_{\lambda_4} (-\sqrt{\pi}) \times d_M^J(\Theta_{34}) D_{\lambda_4, \lambda_6}^{1*}(\Theta_{56}) \sum_{L_{34}, S_{34}(=1)} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \times \lambda_6 \times \alpha_{L_{34}, S_{34}}^{J^{PC}, M} \quad (30)$$

$\alpha_{L_{34},S_{34}}^{J^{PC}}$ is the product of the J^{PC} production- and decay- amplitudes.

The corresponding amplitude in the helicity description for a given J and for a given (λ_1, λ_2) combination (fixed M) is apart from kinematical factors-(see (23)):

$$A_J^{M,\lambda_6} = (2J+1) \sum_{\lambda_4} d_{M,\lambda_4}^J(\Theta_{34}) D_{\lambda_4 \lambda_6 - \lambda_5}^{1*}(\Omega_{56}) \times \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, M | J, M \rangle \times \\ \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | S_{12}, M \rangle \sum_{L_{34}, S_{34}(=1)} \sqrt{\frac{2L_{34}+1}{2J+1}} \langle L_{34}, 0, 1, \lambda_4 | J, \lambda_4 \rangle \times \lambda_6 \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{J^{PC}} \quad (31)$$

The comparison of both expressions for a given $\lambda_1, \lambda_2(M)$ combination and for given L_{34}, S_{34} values yields the correspondance of the amplitudes of the Standard Method and of the helicity description:

$$(-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{J^{PC}, M} = (2J+1) \times \sqrt{\frac{2L_{34}+1}{2J+1}} \sum_{L_{12}, S_{12}} \sqrt{\frac{2L_{12}+1}{2J+1}} \langle L_{12}, 0, S_{12}, M | J, M \rangle \times \\ \times \langle 1/2, \lambda_1, 1/2, -\lambda_2 | S_{12}, M \rangle \times \hat{T}_{L_{12}, S_{12}, L_{34}, S_{34}(=1)}^{J^{PC}} \quad (32)$$

For $M=0$, there exist two combinations (Singlet, $S_{12}=0$; Triplet, $S_{12}=1$), for $M \pm 1$ there is only a triplet state.

Examples are given for $J=1$ and $J=2$:

$J=1$:

$$(-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{1^{+-}, 0} (\text{singlet}) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} \sqrt{1/2} \times \hat{T}_{1,0, L_{34}, S_{34}(=1)}^{1^{+-}} \\ (-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{1^{--}, 0} (\text{triplet}) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} [\sqrt{1/6} \times \hat{T}_{0,1, L_{34}, S_{34}(=1)}^{1^{--}} - \sqrt{1/3} \times \hat{T}_{2,1, L_{34}, S_{34}(=1)}^{1^{--}}] \\ (-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{1^{--}, 1} (\text{triplet}) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} [\sqrt{1/3} \times \hat{T}_{0,1, L_{34}, S_{34}(=1)}^{1^{--}} + \sqrt{1/6} \times \hat{T}_{2,1, L_{34}, S_{34}(=1)}^{1^{--}}] \\ (-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{1^{--}, -1} (\text{triplet}) = 3 \times \sqrt{\frac{2L_{34}+1}{3}} [\sqrt{1/3} \times \hat{T}_{0,1, L_{34}, S_{34}(=1)}^{1^{--}} + \sqrt{1/6} \times \hat{T}_{2,1, L_{34}, S_{34}(=1)}^{1^{--}}] \quad (33)$$

$J=2$:

$$(-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{2^{--}, 1} (\text{triplet}) = 5 \times \sqrt{\frac{2L_{34}+1}{5}} (-\sqrt{1/2}) \times \hat{T}_{2,1, L_{34}, S_{34}(=1)}^{2^{--}}$$

$$(-\sqrt{\pi}) \times \alpha_{L_{34}, S_{34}(=1)}^{2^{--}, -1}(\text{triplett}) = 5 \times \sqrt{\frac{2L_{34} + 1}{5}} \sqrt{1/2} \times \hat{T}_{2,1, L_{34}, S_{34}(=1)}^{2^{--}} \quad (34)$$

Note:Negative sign! for J=2,M=1

These examples show,that the use of Standard Method amplitudes and of helicity amplitudes is not equivalent.The Standard Method amplitudes are linear combinations of the helicity amplitudes,which might be still tolerable.The real difference is in their signs,which causes a problem,when coherent sums over J are constructed.E.g.,the Standard Method amplitudes give wrong ω density matrix elements,which don't fulfill the general symmetry rules(see next chapter).

In addition,the helicity amplitudes are well defined,in contrast to the Standard-Method amplitudes,which are an undefined mixture of production and decay amplitudes and of kinematical factors.Only with helicity amplitudes a proper comparison between reactions at different energies is possible.Therefore,future analyses should be performed with helicity amplitudes using the expression(27).

The formulae here are given for $\bar{p}p \rightarrow \omega\pi^0$,but can be easily extended to particles with different spins,e.g. $f_0, f_2, ..$,which are discussed in [10] and [11].In these cases,for each resonance a separate amplitude has to be constructed,which are then added coherently.All final state particles are added incoherently,as was here the case with the γ .

3 Spin Density Matrix Formulation

In the following the spin-density-matrix(ρ) formalism for the reaction under study is introduced.The fit of ρ -matrix elements is equivalent to the fit of the helicity amplitudes discussed above.However,quite often the ρ -matrix elements are calculated from the fitted helicity amplitudes of a full PWA.Both methods are discussed in the following.On the other hand,several or all of the ρ -matrix elements-averaged over the production process- can be determined from the measured angular distributions of the decaying particle [17].That is discussed for the ω -decay.

3.1 Rho-Matrix Formalism

In (2) $|T_{fi}|^2$ can be rewritten in the following form(isospin factor neglected):

$$\begin{aligned} |T_{fi}|^2 &= \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_5, \lambda_6} \times \sum_{\lambda_4, \lambda_4'} [T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^* \times T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4'} \times A_{\lambda_4, \lambda_5 - \lambda_6}^* \times A_{\lambda_4', \lambda_5 - \lambda_6}] = \\ &= \left\{ \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |T_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 \right\} \times \sum_{\lambda_5, \lambda_6} \times \sum_{\lambda_4, \lambda_4'} [A_{\lambda_4, \lambda_5 - \lambda_6}^* \times \rho_{\lambda_4 \lambda_4'} \times A_{\lambda_4', \lambda_5 - \lambda_6}] = \\ &= \left\{ \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |T_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 \right\} \times \frac{3}{4\pi} \times \sum_{\lambda_5, \lambda_6} \sum_{\lambda_4, \lambda_4'} [D_{\lambda_4, \lambda_5 - \lambda_6}^1(\Omega_{56}) \times \rho_{\lambda_4 \lambda_4'} \times D_{\lambda_4', \lambda_5 - \lambda_6}^{1*}(\Omega_{56}) \times |A_{\lambda_5 \lambda_6}^1|^2] \end{aligned}$$

(35)

with the spin density matrix elements for particle 4 (ω)

$$\rho_{\lambda_4 \lambda_4'} = \frac{1}{\sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |T_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2} \times \sum_{\lambda_1, \lambda_2, \lambda_3} T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}^* \times T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4'} \quad (36)$$

The ρ -matrix is hermitean and has trace=1 by definition. The diagonal elements are real. It has additional symmetries, if e.g. parity is conserved in the production process.

In our case(Spin=1; Parity Conservation) the ρ -matrix has the form (see [12])

$$\rho_{\lambda_4 \lambda_4'}^0 = \begin{pmatrix} 1/2(1 - \rho_{00}^0) & \Re \rho_{10}^0 + i \Im \rho_{10}^0 & \Re \rho_{1-1}^0 \\ \Re \rho_{10}^0 - i \Im \rho_{10}^0 & \rho_{00}^0 & -(\Re \rho_{10}^0 - i \Im \rho_{10}^0) \\ \Re \rho_{1-1}^0 & -(\Re \rho_{10}^0 + i \Im \rho_{10}^0) & 1/2(1 - \rho_{00}^0) \end{pmatrix} \quad (37)$$

with the four independent parameters ρ_{00}^0 , $\Re \rho_{10}^0$, $\Im \rho_{10}^0$ and $\Re \rho_{1-1}^0$. ⁰ refers to measurements with unpolarized particles in the initial state([12]).

No polarization and no alignment for the S=1 case mean: $\rho_{11}^0 = \rho_{-1-1}^0 = \rho_{00}^0 = 1/3$
Alignment means: $\rho_{11}^0 = \rho_{-1-1}^0 \neq \rho_{00}^0$
Polarization means: $\rho_{11}^0 \neq \rho_{-1-1}^0$

The ρ matrix elements are generally dependent on s, $\cos \Theta_{34}$ and Φ_{34} and can be derived -together with the helicity amplitudes $A_{\lambda_5 \lambda_6}^1$ -from the measured data using the following expression based on (1),(12) and (35):

$$\begin{aligned} \frac{d^4 N}{d \cos \Theta_{34} d \cos \Theta_{56} d \Phi_{34} d \Phi_{56}} &= L_{int} \times 1/4 \times 1/2 \times \frac{1}{4s^{1/2} p_{\bar{p}}} \times \frac{1}{2m_{\omega} \Gamma_{\omega}} \times Kin_{34} \times Kin_{56} \times \\ &\times \left\{ \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |T_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2 \right\} \times \frac{3}{4\pi} \sum_{\lambda_5, \lambda_6} \sum_{\lambda_4, \lambda_4'} [D_{\lambda_4, \lambda_5 - \lambda_6}^1(\Omega_{56}) \times \rho_{\lambda_4 \lambda_4'} \times D_{\lambda_4', \lambda_5 - \lambda_6}^{1*}(\Omega_{56}) \times |A_{\lambda_5 \lambda_6}^1|^2] = \\ &= L_{int} \times \frac{1}{2m_{\omega} \Gamma_{\omega}} \times Kin_{56} \times \frac{3}{4\pi} \times \frac{d\sigma}{d\Omega_{34}} \sum_{\lambda_5, \lambda_6} \sum_{\lambda_4, \lambda_4'} [D_{\lambda_4, \lambda_5 - \lambda_6}^1(\Omega_{56}) \times \rho_{\lambda_4 \lambda_4'} \times D_{\lambda_4', \lambda_5 - \lambda_6}^{1*}(\Omega_{56}) \times |A_{\lambda_5 \lambda_6}^1|^2] \end{aligned} \quad (38)$$

with

$$\frac{d\sigma}{d\Omega_{34}} = 1/4 \times 1/2 \times \frac{1}{4s^{1/2} p_{\bar{p}}} \times Kin_{34} \times \sum_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} |T_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}|^2$$

(38) is equivalent to(21)and can be used for the analysis of the complete reaction chain. Here the fit parameters(for fixed s) are no more the \hat{T} -amplitudes, but the

rho-matrix elements for binned $\cos \Theta_{34}$ and binned Φ_{34} values, together with the amplitudes A_{56}^1 . All rho-matrix elements can be determined. The number of fit parameters is not the same as in (21), but in the same ball park. The number will also increase with s.

In many cases the rho-matrix elements are not fitted from (38), but are determined using the $T_{\lambda_1, \lambda_2, \lambda_3, \lambda_4}$ -amplitudes taken from a full PWA-analysis (see (21)). This procedure was used in [13, 14, 15].

Note:

In electroweak interactions the rho-matrix elements can be exactly calculated (see, e.g., [16], pages 372 ff.)

3.2 Averaged Rho-Matrix Formalism

Several elements of ρ -averaged over the production process-can also be derived from the measurement of the angular distribution of the decaying particle 4(ω) only (Schilling's Method) (see [12, 13, 14, 15])

In our case the expression for the averaged 2-dimensional angular distribution is given by (see(38))

$$\begin{aligned} I(\Omega_{56}) &= \frac{dN/N}{d\cos \Theta_{56} d\Phi_{56'}} = \int \frac{dN/N}{d\cos \Theta_{34} d\cos \Theta_{56} d\Phi_{34} d\Phi_{56'}} d\cos \Theta_{34} d\Phi_{34} = \\ &= L_{int} \times \frac{1}{2m_\omega \Gamma_\omega} \times Kin_{56} \times \frac{3}{4\pi} \times \sum_{\lambda_4, \lambda_4'} \times \sum_{\lambda_5, \lambda_6} [D_{\lambda_4, \lambda_5 - \lambda_6}^{1*}(\Omega_{56}) \times \\ &\quad \times \underbrace{\int \frac{d\sigma}{d\Omega_{34}} \times \rho_{\lambda_4 \lambda_4'}^0 d\Omega_{34}}_{\rho_{\lambda_4 \lambda_4'}^0 \times \sigma} \times D_{\lambda_4', \lambda_5 - \lambda_6}^{1*}(\Omega_{56}) \times |A_{\lambda_5 \lambda_6}^1|^2] \times \underbrace{\frac{1}{L_{int} \times \sigma}}_{1/N} \end{aligned} \quad (39)$$

with the averaged ρ -matrix elements

$$\overline{\rho_{\lambda_4 \lambda_4'}^0} \times \sigma = \int \frac{d\sigma}{d\Omega_{34}} \rho_{\lambda_4 \lambda_4'}^0 d\Omega_{34} \quad (40)$$

The expression(38) results to:

$$\begin{aligned} \frac{dN/N}{d\cos \Theta_{56} d\phi_{56'}} &\propto [1/2(1 + \overline{\rho_{00}^0}) + 1/2(1 - 3\overline{\rho_{00}^0})\cos \Theta_{56}^2 + \\ &\quad + \overline{\rho_{1-1}^0}\sin \Theta_{56}^2 \cos 2\phi_{56'} + \sqrt{2}\Re \overline{\rho_{10}^0} \sin 2\Theta_{56} \cos \phi_{56'}] \end{aligned} \quad (41)$$

Note,that only three of the four independent ρ matrix elements can be determined by this method.

The expression for $I(\cos \Theta_{56})$ for the $\omega \rightarrow \pi^0 \gamma$ decay was worked out from $\int I(\Omega_{56}) d\Phi_{56}$ (40) yielding

$$I(\cos \Theta_{56}) = \frac{dN}{d \cos \Theta_{56}} \propto [(1 + \overline{\rho_{00}^0}) + (1 - 3\overline{\rho_{00}^0}) \times \cos \Theta_{56}^2] \quad (42)$$

Here, only diagonal elements of $\overline{\rho^0}$ contribute and can be determined.

The elements of $\overline{\rho^0}$ determined from (36)-averaged over Ω_{34} - and from the fit of angular distributions(40,41) must be identical. This has been demonstrated in [13, 14, 15].

In cases, where not the full reaction chain can be analyzed(too many parameters or incomplete measurements), equations like (40,41) are useful in determining the J^P values of unknown resonances, analyzing their decay distributions. That is particularly efficient, when not only one decay but a decay chain is available [17]. That will be further discussed in a forthcoming note.

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References

- [1] H. Pilkuhn, Relativistic Particle Physics, Springer Verlag
- [2] C. Berger, Teilchenphysik, Eine Einfuehrung, Springer Verlag
- [3] H. Pilkuhn, The Interactions of Hadrons, North Holland
- [4] A.D. Martin and T.D. Spearman, Elementary Particle Theory, North Holland
- [5] C. Voelcker, Thesis, LMU Munich
- [6] S.U. Chung, Spin Formalisms, CERN Yellow Report
- [7] K. Peters, A Primer on Partial Wave Analysis, arXiv:hep-ph/0412069
- [8] J. Richman, An Experimenter's Guide to the Helicity Formalism, CALT-68-1148
- [9] K. Beuchert, Thesis, Univ. Bochum
- [10] T. Degener, Thesis, Univ. Bochum
- [11] J. Luedemann, Thesis, Univ. Bochum

- [12] K. Schilling,P.Seyboth and G.Wolf, Nucl.Phys. B15,(1970),397
- [13] M. Williams et al., arXiv:0908.2910v3
- [14] M. Williams, Thesis,Carnegie Mellon University, 2007
- [15] B. Kopf, Workshop on Amplitude Analysis in Hadron Spectroscopy, Trento
- [16] O.Nachtmann, Elementarteilchphysik, Vieweg-Verlag
- [17] R. Kutschke, An Angular Distribution Cookbook,
<http://home.fnal.gov/~kutschke>