J^P-determination from decay chains,v1

Helmut Koch

July 25, 2011

Abstract

$\overline{p}p$ reactions in flight have been analyzed so far using a full PWA formalism.This method is restricted to few particles in the final state and to moderate $\overline{p}$ energies.In PANDA reactions at high energies will be studied,resulting in an increase of the number of final state particles and of amplitudes to be determined.In such cases a different approach presented here may be more successfull.A resonance with given invariant mass is reconstructed from the data using its decay(decay chain).The rest of the event (X) is treated in a global manner using an averaged spin density matrix for its spin structure.From the measured angular decay distributions of the resonance its J^P values can be determined or constrained.The method is restricted to isolated resonances,which are expected to appear more often at higher energies.Many of the recent analyses concerning exotic particles at BaBar and Belle were performed using this formalism.The method developped here is based on a note of R. Kutschke.The determination of the J^P value of $D_{s1}(2536)$ is discussed in detail as example.

1 Definition of the averaged spin density matrix

In a preceeding note [1],the analysis of a complete $\overline{p}p$-reaction in flight was discussed for known J^P values of the particles involved.The amplitude depends critically on these values,particularly the angle dependent part. Here,the determination of unknown J^P values of particles within a decay chain is discussed.It is most efficient, if the J^P values of all particles except one are known.This is for instance the case in the decay chain $D_{s1}^+(2536) \rightarrow D^{*+}(2010) + K_s^0$, $D^{*+}(2010) \rightarrow D^0 + \pi^+$,which will be discussed in detail

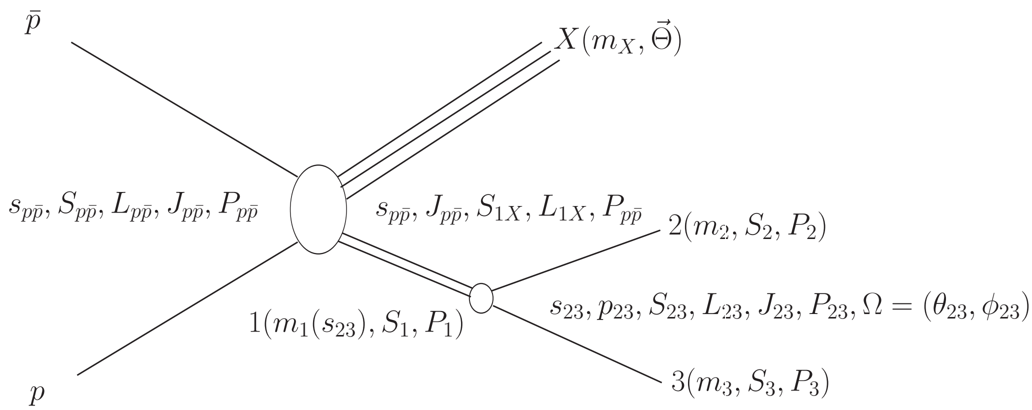


Figure 1: Definitions for the reaction $\overline{p}p \to X + 1; 1 \to 2 + 3$
m=mass of the particle;s=CM-energy squared;S=spin;L=orbital angular momentum;J=total angular momentum;P=parity

later. The J^P values of $D^{*+}(2010)$, D^0, K_s^0 and $\pi^+$are known. The measured angular distributions of the $D_{s1}^+(2536)$- and $D^{*+}(2010)$-decays give information on $J^P(D_{s1}^+(2536))$. $D_{s1}^+(2536)$is produced from a process,globally described by an averaged spin density matrix ($\overline{\rho}$). The situation is sketched in Figure 1. The full transition amplitude $A(\overline{p}p \to X + 1; 1 \to 2 + 3)$ can be divided [2] in a production part for particle 1,described by $\overline{\rho}$, and the decay amplitude of particle 1, A($\mathit{\Omega}$;λ_1,λ_2,λ_3).

$$A(\overline{p}p \to X + 1; 1 \to 2 + 3) = \sum_{\lambda_1} P(\vec{\Theta}, M, \lambda_1) \times A(\mathit{\Omega}; \lambda_1, \lambda_2, \lambda_3) \qquad (1)$$

with P$(\vec{\Theta}, M, \lambda_1)$ as production amplitude of the particles 1 + X (including X decay),$\vec{\Theta}$ describing all continuous variables (momenta,angles)of the production process and M describing all helicities to be added incoherently.

The **differential cross section** is given by

$$\begin{aligned} d\sigma &= flux \times \left|A(\overline{p}p \to X + 1; 1 \to 2 + 3)\right|^2 \times dLips(s_{\overline{p}p}, \vec{\Theta}, p_2, p_3) = \\ &= flux \times \sum_{\lambda_1, \lambda_1', \lambda_2, \lambda_3, M} [P(\vec{\Theta}, M, \lambda_1) A(\mathit{\Omega}; \lambda_1, \lambda_2, \lambda_3)]^* [P(\vec{\Theta}, M, \lambda_1') A(\mathit{\Omega}; \lambda_1', \lambda_2, \lambda_3)] \times \\ &\times dLips(s, \vec{\Theta}) \times \frac{ds_1}{2\pi} \frac{dLips(m_1, p_2, p_3)}{(m_1^2 - s_1)^2 + m_1^2 \Gamma_1^2} \qquad \text{(see [3, 4])} \end{aligned} \qquad (2)$$

with m_1 and Γ_1= nominal mass and width of particle 1, and $s_1 = s_{23}$=invariant mass squared of particles 2 and 3.

The differential cross section,integrated over X,is

$$d'\sigma = \int \frac{d\sigma}{dLips(s,\vec{\Theta})} \times dLips(s,\vec{\Theta}) =$$
$$= \sigma \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_3} A^*(\Omega;\lambda_1,\lambda_2,\lambda_3) \times \overline{\rho_{\lambda_1,\lambda_1'}} \times A(\Omega;\lambda_1',\lambda_2,\lambda_3) \times$$
$$\times \frac{ds_1}{2\pi} \times \frac{dLips(m_1,p_2,p_3)}{(m_1^2 - s_1)^2 + m_1^2\Gamma_1^2} = \tag{3}$$

$$\stackrel{m_1\Gamma_1 \to 0}{=} \sigma \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_3} A^*(\Omega;\lambda_1,\lambda_2,\lambda_3) \times \overline{\rho_{\lambda_1,\lambda_1'}} \times A(\Omega;\lambda_1',\lambda_2,\lambda_3) \times$$
$$\times \frac{dLips(m_1,p_2,p_3)}{2m_1\Gamma_1} \qquad \text{(see [1])}$$

with the ρ-matrix elements,averaged over the production process

$$\overline{\rho_{\lambda_1,\lambda_1'}} = \frac{1}{\sigma} \int \sum_M P^*(\vec{\Theta};M,\lambda_1) P(\vec{\Theta};M,\lambda_1') dLips(s,\vec{\Theta}) \tag{4}$$

and

$$\sigma = flux \times \int \sum_{M,\lambda_1} P^*(\vec{\Theta};M,\lambda_1) P(\vec{\Theta};M,\lambda_1) dLips(s,\vec{\Theta}) \tag{5}$$

The **number of reacting particles dN** is given by

$$dN = L_{int} \times d'\sigma =$$
$$= N \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_3} A^*(\Omega;\lambda_1,\lambda_2,\lambda_3) \times \overline{\rho_{\lambda_1,\lambda_1'}} \times A(\Omega;\lambda_1',\lambda_2,\lambda_3) \times$$
$$\times \frac{dLips(m_1,p_2,p_3)}{2m_1\Gamma_1} \qquad \text{with} \quad N = \sigma \times L_{int};\ L_{int} = integrated luminosity. \tag{6}$$

Here it is assumed that $\overline{p}$ and p are unpolarized.Then $\overline{\rho}$ means $\overline{\rho^0}$(for details see [5]).

For reactions with subsequent decays,for instance $\overline{p}p \rightarrow X + 1; 1 \rightarrow 2 + 3; 2 \rightarrow 4 + 5$ (see Fig.2) (6) reads:

$$dN = N \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_2',\lambda_3,\lambda_4,\lambda_5} A^*(\Omega;\lambda_1,\lambda_2,\lambda_3) \times \overline{\rho_{\lambda_1,\lambda_1'}} \times A(\Omega;\lambda_1',\lambda_2,\lambda_3) \times$$
$$\times B^*(\Omega';\lambda_2,\lambda_4,\lambda_5) B(\Omega';\lambda_2',\lambda_4,\lambda_5) \times \frac{dLips(m_1,p_2,p_3)}{2m_1\Gamma_1} \frac{dLips(m_2,p_4,p_5)}{2m_2\Gamma_2} \tag{7}$$

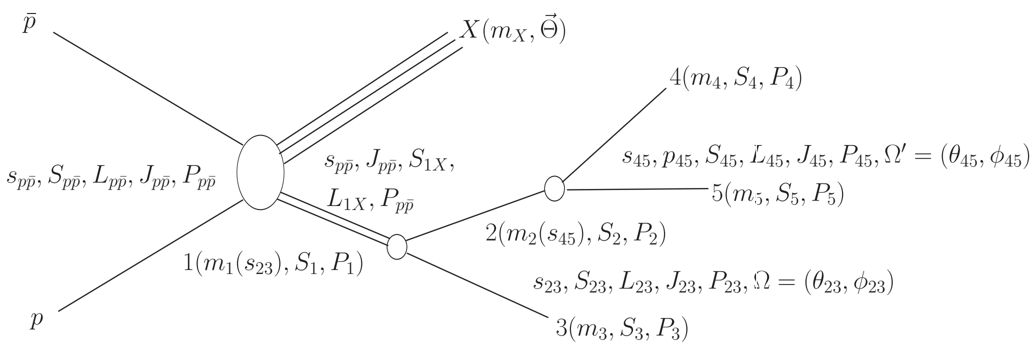


Figure 2: Definitions for the reaction $\overline{p}p \rightarrow X + 1; 1 \rightarrow 2 + 3; 2 \rightarrow 4 + 5$, Nomenclature as in Fig.1

Often it is useful,not to use $\cos\Theta, \Phi, \cos\Theta'$ and Φ',but instead $\cos\Theta, \Phi, \cos\Theta'$ and χ with $\chi = \Phi' - \Phi$ as the relative angle between the decay planes of the particles 1 and 2.
The formulae for the differential cross section and the angular distributions stay the same.

2 Rho-Matrices for different spins

The elements of the ρ-matrix are hermitian by definition,the diagonal elements are real and have trace 1.If spatial parity in the production process is conserved,the ρ matrix elements fulfill the relations

$$\rho_{\lambda_1\lambda_1'} = (-1)^{\lambda_1-\lambda_1'} \rho_{-\lambda_1-\lambda_1'} \tag{8}$$

The values of the ρ-matrix elements are dependent on the reference system.For the CM-System of particle 1(z-axis along the flight direction of 1;y-axis in the production plane of 1) and in the case,that parity is conserved in the production process of 1,the matrix elements have the following form [5, 3, 4]:

$S_1 = 0$: $\qquad \rho = 1$

$$S_1 = 1/2: \quad \rho = \begin{pmatrix} \Re\rho_{1/2\,1/2} & \Im\rho_{1/2\,-1/2} \\ -\Im\rho_{1/2\,-1/2} & \Re\rho_{1/2\,1/2} \end{pmatrix} \quad \text{with } 2\Re\rho_{1/2\,1/2} = 1 \text{ (1 indep. parameter)}$$

$\Big($ Often the ρ-matrix for $S_1 = 1/2$ is written in form of $\sigma - matrices$:
$\rho = 1/2 \times (1 + \vec{P}\vec{\sigma}) = 1/2 \times \left(\begin{matrix} 1 & -\imath P_y \\ \imath P_y & 1 \end{matrix} \right)\Big)$

$$S_1 = 1: \quad \rho = \begin{pmatrix} \Re\rho_{11} & \rho_{10} & \Re\rho_{1-1} \\ \rho_{10}^* & \Re\rho_{00} & -\rho_{10}^* \\ \Re\rho_{1-1} & -\rho_{10} & \Re\rho_{11} \end{pmatrix} \text{with } 2\Re\rho_{11} + \Re\rho_{00} = 1 \quad \text{(4 indep par.)}$$

$$S_1 = 3/2: \quad \rho = \begin{pmatrix} \Re\rho_{33} & \rho_{31} & \rho_{3-1} & \Im\rho_{3-3} \\ \rho_{31}^* & \Re\rho_{11} & \Im\rho_{1-1} & \rho_{3-1}^* \\ \rho_{3-1}^* & -\Im\rho_{1-1} & \Re\rho_{11} & -\rho_{31}^* \\ \Im\rho_{3-3} & \rho_{3-1} & -\rho_{31} & \Re\rho_{33} \end{pmatrix} \text{with } 2\Re\rho_{33} + 2\Re\rho_{11} = 1 \quad \text{(8 indep. par.)}$$

$$S_1 = 2: \qquad \rho = \begin{pmatrix} \Re\rho_{22} & \rho_{21} & \rho_{20} & \rho_{2-1} & \Re\rho_{2-2} \\ \rho_{21}^* & \Re\rho_{11} & \rho_{10} & \Re\rho_{1-1} & -\rho_{2-1}^* \\ \rho_{20}^* & \rho_{10}^* & \Re\rho_{00} & -\rho_{10}^* & \rho_{20}^* \\ \rho_{2-1}^* & \Re\rho_{1-1} & -\rho_{10} & \Re\rho_{11} & -\rho_{21}^* \\ \Re\rho_{2-2} & -\rho_{2-1} & \rho_{20} & -\rho_{21} & \Re\rho_{22} \end{pmatrix}$$

with $2\Re\rho_{22} + 2\Re\rho_{11} + \Re\rho_{00} = 1$ (12 indep. parameters)

It is obvious,that the number of independent parameters of the ρ-matrices increases with the spin of the particles.For a J^P-determination from the measured angular distributions often only the diagonal matrix elements are needed(see later example),which restricts the number of independent parameters drastically,e.g. to one for a spin 1 particle,or to two for a spin 2 particle.

3 Angular distributions

For the decay chain $\overline{p}p \to X + 1; 1 \to 2 + 3; 2 \to 4 + 5$ the combined angular distribution $I(\Omega, \Omega')$ is given by

$$
\begin{aligned}
I(\Omega,\Omega') = \frac{\frac{dN}{N}}{d\Omega d\Omega'} = & \\
& = Kin_{23} \times Kin_{45} \times \frac{1}{2m_1\Gamma_1} \times \frac{1}{2m_2\Gamma_2} \times \frac{2S_1+1}{4\pi} \times \frac{2S_2+1}{4\pi} \times \\
& \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_2',\lambda_3,\lambda_4,\lambda_5} \rho_{\lambda_1\lambda_1'} (D^{S_1}_{\lambda_1,\lambda_2-\lambda_3}(\Omega) A^*_{\lambda_2,\lambda_3} D^{S_1*}_{\lambda_1',\lambda_2'-\lambda_3}(\Omega) A^*_{\lambda_2',\lambda_3}) \times \\
& \times (D^{S_2}_{\lambda_2,\lambda_4-\lambda_5}(\Omega') B^*_{\lambda_4,\lambda_5} D^{S_2*}_{\lambda_2',\lambda_4-\lambda_5}(\Omega') B_{\lambda_4),\lambda_5})
\end{aligned}
\tag{9}
$$

using (7) and

$$
\begin{aligned}
dLips(m_1,p_2,p_3) &= \frac{p_{23}}{16\pi^2 m_1} \times d\Omega = Kin_{23} \times d\Omega \qquad \text{and} \\
dLips(m_2,p_4,p_5) &= \frac{p_{45}}{16\pi^2 m_2} \times d\Omega' = Kin_{45} \times d\Omega'
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
I(\Omega,\Omega') = & \frac{1}{\sum_{\lambda_2,\lambda_3} \left|A_{\lambda_2\lambda_3}\right|^2} \times \frac{1}{\sum_{\lambda_4,\lambda_5} \left|A_{\lambda_4\lambda_5}\right|^2} \times \frac{2S_1+1}{4\pi} \times \frac{2S_2+1}{4\pi} \times \\
& \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_2',\lambda_3,\lambda_4,\lambda_5} \rho_{\lambda_1\lambda_1'} (D^{S_1}_{\lambda_1,\lambda_2-\lambda_3}(\Omega) A^*_{\lambda_2,\lambda_3} D^{S_1*}_{\lambda_1',\lambda_2'-\lambda_3}(\Omega) A_{\lambda_2',\lambda_3}) \times \\
& \times (D^{S_2}_{\lambda_2,\lambda_4-\lambda_5}(\Omega') B^*_{\lambda_4,\lambda_5} D^{S_2*}_{\lambda_2',\lambda_4-\lambda_5}(\Omega') B_{\lambda_4,\lambda_5})
\end{aligned}
\tag{11}
$$

with $\quad \Gamma_1 = \dfrac{1}{2m_1} Kin_{23} \sum_{\lambda_2,\lambda_3} \left|A_{\lambda_2\lambda_3}\right|^2; \quad \Gamma_2 = \dfrac{1}{2m_2} Kin_{45} \sum_{\lambda_4,\lambda_5} \left|A_{\lambda_4\lambda_5}\right|^2 \qquad (12)$

Note:(11) is identical to (31) in [2] with the definition $\Gamma_1 = \sum_{\lambda_2,\lambda_3} \left|A_{\lambda_2\lambda_3}\right|^2$ which is used in [2],in contrast to the definition used here(12).Reference[2] absorbs the kinematical parts of (12) in the definition of the helicity amplitudes.Its helicity amplitudes have the dimension [$\sqrt{GeV}$],while ours have the dimension [GeV]

Integrating (11) over Ω or Ω',one obtains the single angular distributions

$$I(\Omega) = \int I(\Omega,\Omega')d\Omega' = \frac{1}{\sum_{\lambda_2,\lambda_3}\left|A_{\lambda_2\lambda_3}\right|^2} \times \frac{2S_1+1}{4\pi} \times \\ \times \sum_{\lambda_1,\lambda_1',\lambda_2,\lambda_3} \overline{\rho_{\lambda_1\lambda_1'}}(D^{S_1}_{\lambda_1,\lambda_2-\lambda_3}(\Omega)A^*_{\lambda_2,\lambda_3}D^{S_1*}_{\lambda_1',\lambda_2-\lambda_3}(\Omega)A_{\lambda_2,\lambda_3}) \tag{13}$$

$$I(\Omega') = \int I(\Omega,\Omega')d\Omega = \frac{1}{\sum_{\lambda_2,\lambda_3}\left|A_{\lambda_2\lambda_3}\right|^2}\frac{1}{\sum_{\lambda_4,\lambda_5}\left|A_{\lambda_4\lambda_5}\right|^2}\frac{2S_2+1}{4\pi} \times \\ \times \sum_{\lambda_2,\lambda_3,\lambda_4,\lambda_5}\left|d^{S_2}_{\lambda_2,\lambda_4-\lambda_5}\right|^2\left|A_{\lambda_2\lambda_3}\right|^2\left|B_{\lambda_4\lambda_5}\right|^2 \tag{14}$$

Note:$I(\Omega')$ does not depend on Φ'.

Important point:$I(\Omega')$ is independent on the production process of particle $1,\overline{\rho}$ does not appear.This is the reason,why most J^P analysis start with this distribution.

Integration of (11) over $\Phi,\Phi'(\chi')$ delivers the following distribution:

$$I(\Theta,\Theta') = \frac{1}{\sum_{\lambda_2,\lambda_3}\left|A_{\lambda_2\lambda_3}\right|^2} \times \frac{1}{\sum_{\lambda_4,\lambda_5}\left|A_{\lambda_4\lambda_5}\right|^2} \times \frac{2S_1+1}{2} \times \frac{2S_2+1}{2} \times \\ \times \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5} \rho_{\lambda_1\lambda_1}\left|d^{S_1}_{\lambda_1,\lambda_2-\lambda_3}(\Omega)\right|^2\left|A_{\lambda_2,\lambda_3}\right|^2 \times \left|d^{S_2}_{\lambda_2,\lambda_4-\lambda_5}(\Omega')\right|^2 \times \left|B_{\lambda_4,\lambda_5}\right|^2 \tag{15}$$

Integration of (15) over Θ or Θ' delivers the distributions:

$$I(\Theta) = \frac{1}{\sum_{\lambda_2,\lambda_3}\left|A_{\lambda_2\lambda_3}\right|^2}\frac{2S_1+1}{2}\sum_{\lambda_1,\lambda_2,\lambda_3}\rho_{\lambda_1\lambda_1}\left|(d^{S_1}_{\lambda_1,\lambda_2-\lambda_3}(\Omega)\right|^2\left|A_{\lambda_2,\lambda_3}\right|^2 \tag{16}$$

$$I(\Theta') = \frac{1}{\sum_{\lambda_2,\lambda_3}\left|A_{\lambda_2\lambda_3}\right|^2}\frac{1}{\sum_{\lambda_4,\lambda_5}\left|A_{\lambda_4\lambda_5}\right|^2}\frac{2S_2+1}{2}\sum_{\lambda_2,\lambda_3,\lambda_4,\lambda_5}\left|A_{\lambda_2,\lambda_3}\right|^2\left|d^{S_2}_{\lambda_2,\lambda_4-\lambda_5}(\Omega')\right|^2\left|B_{\lambda_4,\lambda_5}\right|^2 \tag{17}$$

Note:Integration over Φ eliminates the off-diagonal elements of the ρ-matrix.Thus,the $I(\Theta,\Theta')$- and $I(\Theta)$- distributions are only dependent on the diagonal matrix elements.That reduces considerably the number of parameters,which need to be fitted.Again,I(Θ') is independent of the rho-matrix,like in the case of I(Ω').

CAVEATS:
1.$I(\Theta')$ was determined by integration of $I(\Theta,\Theta')$ over Θ from 0 to π.This tacitly assumes,that $I(\Theta)$ has a flat efficiency,which is not always the case.If the efficiency is only flat for instance for $\pi/2 > \Theta > 0$,then one can check,if formula (18) still holds for this range of Θ values.That is often the case.Then for the $I(\Theta')$ distribution only data with the cut $\pi/2 > \Theta > 0$ can be used.
2.It is not excluded,that parts of the X-decay interfere with the decay chain $1 \to 2+3, 2 \to 4+5$.To check this,the data can be arranged according to different Θ values.If in all cases the same $I(\Theta')$ distribution results,no interference terms exist. Both these cases have been addressed in detail in [7].

Note also the subtle definition of the helicity angles Θ, Θ', Φ and $\Phi'(\chi')$.They are defined in the cm-system of the preceeding particle,with the quantization axis z in the flight direction of the particle.This is the price,one has to pay for the simplicity of the helicity formalism.This definition makes the connection between the spin of particle 1 and the angular distribution of particle 2.A nice way to evaluate these angles in a Lorentz invariant manner is given in [6].The situation is depicted in Fig.3 for a special decay chain.

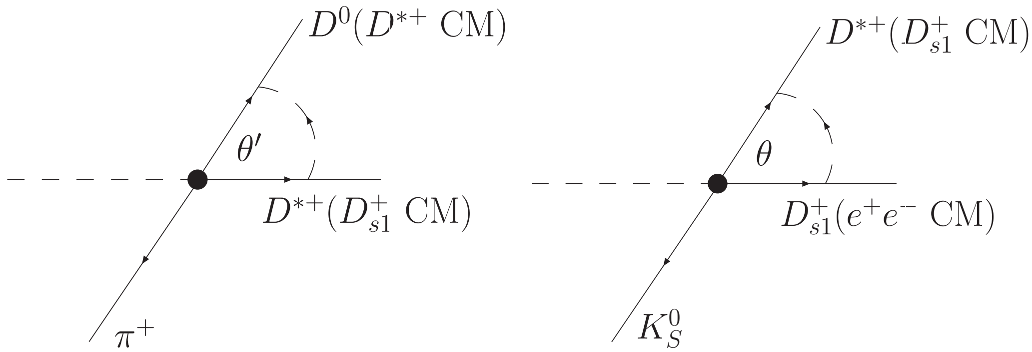


Figure 3: Definition of helicity angles for the decay chain $D^+_{s1}(2536) \to D^{*+} + K^0_s; D^{*+} \to D^0 + \pi^+$

In many cases the $I(\Theta')$ distribution is used to get information on the spins and parities of the particles involved.It depends on the spin of particle 2,but also (through the definition of the helicity angles) on the spin of particle

1.Usually,not all helicity amplitudes (or alternatively LS-amplitudes) can be determined in this way (see example below).
It is more effective,to use the combined angular distributions (11) of both decays.Here,the production process of particle 1,described by its averaged spin density matrix,and the helicity amplitudes of the subsequent decays are fitted simultaneously.This method is most effective in determining the J^P values of the particles involved,as will be demonstrated in the following example.Generally,a chain with many spin 0 particles is preferable,because then only one additional helicity amplitude appears,which can be easily absorbed in the other helicity amplitudes.

4 Analysis of the reaction $e^+e^- \to X + D_{s1}(2536)^+$; $D_{s1}(2536)^+ \to D^{*+} + K_s^0$; $D^{*+} \to D^0 + \pi^+$

The analysis presented here as an example is described in detail in [8]and [7]. The decay chain is $?^? \to 1^- + 0^-; 1^- \to 0^- + 0^-$.The J^P values of all particles involved are known,except the ones of D_{s1}.The task is to determine J^P of D_{s1}(or limit the range of values)from an analysis of the measured angular distributions.
The most efficient way to achieve that goal is the measurement and analysis of the combined angular distributions as given by (57) in [2] and (3) in[8],derived from (11).Under the assumption of $J^P(D_{s1}) = 1^+, 2^-, 3^+$,e.g., the angular distribution is given by

$$I(\Theta,\Theta',\chi) = \int I(\Theta,\Theta',\Phi,\chi)d\Phi \propto \Big(\big|A_{10}\big|^2 \frac{\sin\Theta'^2}{4}\big[\big(1+\cos\Theta^2\big) + \rho_{00}\big(1-3\cos\Theta^2\big)\big]$$
$$+ \big|A_{00}\big|^2 \frac{\cos\Theta'^2}{2}\big[\big(1-\cos\Theta^2\big) - \rho_{00}\big(1-3\cos\Theta^2\big)\big]$$
$$- \frac{1}{4}\big|A_{10}\big|^2\big(1-3\rho_{00}\big)\sin\Theta^2\sin\Theta'^2\cos(2\chi)$$
$$+ \Re(A_{10}^*A_{00})\big(1-3\rho_{00}\big)\sin\Theta\cos\Theta\sin\Theta'\cos\Theta'\cos\chi\Big) \tag{18}$$

A_{10} and A_{00} are the helicity amplitudes of the $D_{s1} \to D^* + K_s^0$ decay.
Here,an integration over Φ has been performed,resulting in the disappearence of non-diagonal rho-matrix elements.Because of parity conservation and trace =1,only one element remains(ρ_{00}).ρ_{00} in this formula is always understood as the averaged value $\overline{\rho_{00}}$
The formula was confirmed in an independent calculation with Mathematica

[9]

The angular distributions for alternative J^Pvalues of $D_s(2536)$,integrated over Φ, are given below [9].

$1^- \rightarrow 1^- + 0^-; 1^- \rightarrow 0^- + 0^-$:

$$I(\Theta,\Theta',\chi) \propto \left|A_{10}\right|^2 \sin\Theta'^2(1+\cos\Theta^2+(1-3\cos(2\chi)+3(-1+\cos(2\chi))\times \times\cos\Theta^2)\rho_{00}+\cos 2\chi\sin\Theta^2) \tag{19}$$

$0^- \rightarrow 1^- + 0^- \rightarrow 0^- + 0^-$:

$$I(\Theta,\Theta',\chi) \propto \rho_{00}(2\times\left|A_{00}\right|^2\cos\Theta^2\cos\Theta'^2+2\left|A_{10}\right|^2\cos\chi^2\sin\Theta^2\sin\Theta'^2- -\Re(A_{10}A_{00}^*)\cos\chi\sin(2\Theta)\sin(2\Theta')) \tag{20}$$

Note:When one is interested in details of the production process, further elements of the rho-matrix can be determined from the four dimensional distribution $I(\Theta,\Theta',\Phi,\chi)$.This is shown for an example in the Appendix A.

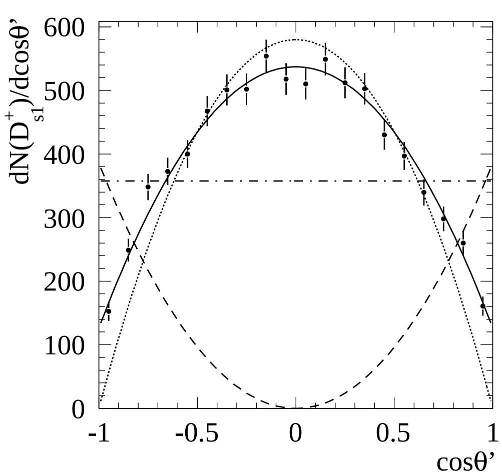
The three dimensional angular distribution measured by Belle was fitted using (18).The determination of eventual background is not trivial for such a distribution. For details see [8].

The result of the fit is
$A_{10}/A_{00} = \sqrt{3.6\pm0.3\pm0.1}\times exp(\pm\imath(1.27\pm0.15\pm0.05)$ and
$\rho_{00} = 0.49\pm0.12\pm0.004$
Note,that not only $\left|A_{10}\right|/\left|A_{00}\right|$ can be determined,but also their relative phase.
Converting the helicity amplitudes to LS-amplitudes yields
D/S = $(0.63\pm0.07\pm0.02)\times exp(\pm\imath(0.76\pm0.03\pm0.01)$.

The BaBar analysis used only the one-dimensional distributions I(Θ') and I(Θ)(see(16,17)).
I(Θ')-distribution:
The distribution was worked out for different assumptions on $J^P(D_{s1}(2536))$(Tab.1).Fig.4 gives the measured I(Θ') distribution,together with the predicted curves.The case $(1^+,2^-,3^+)$(S-,D-wave) is clearly favored,with 1^+ being the most probable value(see χ^2-values in Table 1).$|A_{00}|/|A_{10}|$ was determined to be 0.23 $\pm$0.03,in agreement with Belle.The phase could not be determined.

J^P	$dN(D_{s1}^+)/d\cos\theta'$	$\chi^2/NDF(K4\pi)$
0^+	forbidden	–
0^-	$a\cos^2\theta'$	2142.7/19
$1^-,2^+,3^-,\ldots$	$a\sin^2\theta'$	103.2/19
$1^+,2^-,3^+,\ldots$ (S-wave only)	const	392.1/19
$1^+,2^-,3^+,\ldots$ (S-, D-wave)	$a(\sin^2\theta'+\beta\cos^2\theta')$	24.9/18 ($\beta=0.23\pm0.03$)

Table 1: List of spin-parity values J^P for the D_{s1}^+ and the corresponding decay angle distributions $I(\Theta')$ for the D^{*+}. Under the assumption of a strong decay, 0^+ is forbidden.



.

Figure 4: Efficiency-corrected signal yield as function of $\cos\Theta'$ in data.The following models are fitted to the distribution:$\propto$ $(\sin\Theta'^2+\beta\cos\Theta'^2)$(solid line); $\propto$ constant(dash-dotted line); $\propto\cos\Theta'^2$(dashed line); $\propto\sin\Theta'^2$(dotted line) (from [7])

I(Θ)-distribution:
Assuming $J^P = 1^+, 2^-, 3^+$,I(Θ) is given by (see(16))

$$I(\Theta) \propto (1+\rho_{00})\left|A_{10}\right|^2+(1-\rho_{00})\left|A_{00}\right|^2+(1-3\rho_{00})(\left|A_{10}\right|^2-\left|A_{00}\right|^2)\cos\Theta^2 \quad (21)$$

The measured distribution is given in Fig.5,together with a fit,yielding $\rho_{00} = 0.48 \pm 0.03$, with $\left|A_{00}\right|/\left|A_{10}\right|$ taken from the I(Θ')-distribution.This value is also in agreement with Belle.

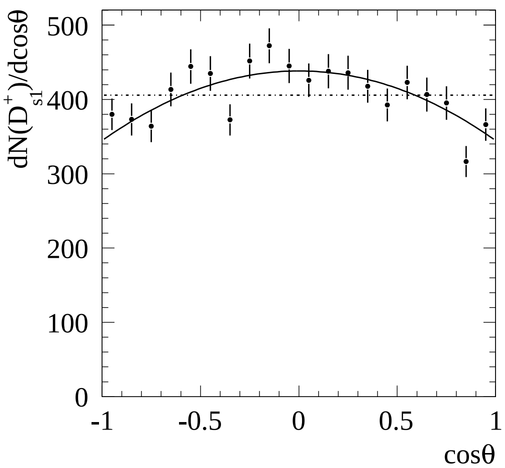


Figure 5: Efficiency corrected signal yield as function of $cos\Theta$ in data.The following models are fitted to the distribution:$\propto$ constant (dotted line);$\propto$ $(1 + t\cos\Theta^2)$(solid line) (from [7])

This example shows,how much information can be obtained on J^P-values of particles in a decay chain using combined or single angular distributions.The combined distributions yield generally more information and should be preferred in an analysis.Many more examples for angular distributions are discussed in [2].

5 Acknowledgement

For discussions and help I thank Dr.B.Kopf(Ruhr-University Bochum).Prof.Dr.P.Kroll(Wuppertal University),B.sc.J.Pychy(Ruhr-University Bochum) and Dr.T.Schroeder(Ruhr University Bochum).

References

[1] H. Koch,Helicity amplitude for $\overline{p}p \to \omega\pi^0, \omega \to \pi^0\gamma$,Panda PWA note

[2] R. Kutschke,An Angular Distribution Cookbook,http://home.fnal.gov/ kutschke

[3] H. Pilkuhn,The Interaction of Hadrons,North Holland

[4] H. Pilkuhn,Relativistic Particle Physics,Springer Verlag

[5] K. Schilling,P.Seyboth and G. Wolf,Nucl.Phys.B15(1970)397

[6] A. Ryd et al.,EvtGen,A Monte Carlo Generator for B-Physics,EvtGen V00-10-07

[7] P. Lees et al.,P.R.D.83,072003(2011)

[8] V. Balagura et al.,P.R.D77,032001(2008)

[9] J. Pychy,Private Communication

A Four-dimensional angular distribution for $D_{s1}(2536)^+ \to D^{*+} + K_s^0; D^{*+} \to D^0 + \pi^+$

The four-dimensional angular distribution for $D_{s1}(2536)^+ \to D^{*+}+K_s^0; D^{*+} \to D^0 + \pi^+$ was worked out using Mathematica [9].It is given below:

$$
\begin{aligned}
I(\Omega,\Omega') \propto\ & 1/2 \times \left|A_{00}\right|^2 \cos\Theta'^2 \sin\Theta^2 + \\
& + 1/8 \times \left|A_{10}\right|^2 \sin\Theta'^2 [3 - \cos(2\chi) - (1+\cos(2\chi))\cos(2\Theta)] + \\
& + \Re(A_{10}A_{00}^*) \cos\chi \cos\Theta \cos\Theta' \sin\Theta \sin\Theta' + \\
& + \Re\rho_{10}[\sqrt{2} \times \left|A_{00}\right|^2 \cos\Theta'^2 \sin(2\Theta)\cos\Phi + \frac{1}{\sqrt{2}}\left|A_{10}\right|^2 \sin\Theta'^2 \sin(2\Theta)\cos\Phi + \\
& + \sqrt{2} \times \left|A_{10}\right|^2 \sin\Theta \sin\Theta'^2(-\cos(2\chi)\cos\Theta\cos\Phi + \sin(2\chi)\sin\Phi) + \\
& + 2\sqrt{2} \times \Re(A_{10}A_{00}^*) \cos\Theta' \sin\Theta'(\cos\chi\cos(2\Theta)\cos\Phi - \cos\Theta\sin\chi\sin\Phi)] + \\
& + \Im\rho_{10}[\sqrt{2} \times \Im(A_{10}A_{00}^*) \sin(2\Theta')(\cos\chi\cos\Phi - \cos\Theta\sin\chi\sin\Theta] + \\
& + \Re\rho_{1-1}[-1/2 \times \Re(A_{10}A_{00}^*) \sin(2\Theta')(\cos\chi\cos(2\Phi)\sin(2\Theta) - 2\sin\chi\sin\Theta\sin(2\Phi))] + \\
& + \rho_{00}[\left|A_{00}\right|^2(-1+3\cos\Theta^2)\cos\Theta'^2 + 1/4 \times \left|A_{10}\right|^2(1-3\cos\Theta^2)\sin\Theta'^2 - \\
& - 3/4 \times \left|A_{10}\right|^2 \cos(2\chi)(-1+\cos\Theta^2)\sin\Theta'^2 - 3\Re(A_{10}A_{00}^*)\cos\chi\cos\Theta\cos\Theta'\sin\Theta\sin\Theta']
\end{aligned}
\tag{22}
$$

It shows,that all elements of the ρ-matrix can be determined from multidimensional angular distributions.(18) is the integral of this expression over Φ.