Notes de travail
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Why the effective form factor is equivalent to $|G_E| = |G_M|$

The expression of the hadron electromagnetic current for the $\bar{p}p$ annihilation in two leptons is derived assuming one photon exchange (OPE):

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2\beta s}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta|G_E|^2\right],\ \beta = \sqrt{1-\frac{1}{\tau}},\ \tau = \frac{s}{4m^2} \tag{1}$$

We will use instead the following expression of the cross section:

$$\frac{d\sigma}{d\cos\theta} = \sigma_0\left[1 + \mathcal{A}\cos^2\theta\right], \tag{2}$$

where σ_0 is the value of the differential cross section at $\theta = \pi/2$ and $\mathcal{A}$ is an angular asymmetry which can be written as a function of the FFs:

$$\sigma_0 = \frac{\pi\alpha^2}{2\beta s}\left(|G_M|^2 + \frac{1}{\tau}|G_E|^2\right)\ \mathcal{A} = \frac{\tau|G_M|^2 - |G_E|^2}{\tau|G_M|^2 + |G_E|^2} = \frac{\tau - \mathcal{R}^2}{\tau + \mathcal{R}^2}. \tag{3}$$

where $\mathcal{R} = |G_E|/|G_M|$. Note that in real experiments, the angular range where the measurement can be performed is usually restricted to $|\cos\theta| \le \bar{c}$ with $\bar{c} = \cos\theta_{max}$. The integrated experimental cross section is

$$\sigma_{int} = \int_{-\bar{c}}^{\bar{c}} \frac{d\sigma}{d\cos\theta} d\cos\theta = \frac{2}{\sigma_0}\,\bar{c}\left(1 + \frac{\mathcal{A}}{3}\bar{c}^2\right) = \frac{\pi\alpha^2}{2\beta s}\bar{c}\left[\left(1+\frac{\bar{c}^2}{3}\right)|G_M|^2 + \frac{1}{\tau}\left(1-\frac{\bar{c}^2}{3}\right)|G_E|^2\right], \tag{4}$$

The total cross section corresponds to $\bar{c} = 1$:

$$\sigma_{tot} = 2\sigma_0\left(1+\frac{\mathcal{A}}{3}\right) = \frac{2\pi\alpha^2}{3\beta s}\left(2|G_M|^2 + \frac{|G_E|^2}{\tau}\right). \tag{5}$$

Definition: One can define an effective form factor from the **total** cross section by

$$|F_p|^2 = \frac{3\beta s \sigma_{tot}}{2\pi\alpha^2 \left(2 + \frac{1}{\tau}\right)}. \tag{6}$$

or from the **integrated** cross section by

$$|F_p|^2 = \frac{\beta s}{\pi\alpha^2} \frac{\sigma_{int}}{\bar{c}\left[\left(1 + \frac{\bar{c}^2}{3}\right) + \frac{1}{\tau}\left(1 - \frac{\bar{c}^2}{3}\right)\right]}. \tag{7}$$

From the experiment, the effective form factor is extracted from the cross section assuming $|G_E| = |G_M|$.

Comparing Eqs. (5) and (6), one can see that in order to compare theory with experiment, the experimental form factor is equivalent to the effective form factor assuming $|G_E| = |G_M|$.