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twoLeptonGen: a Monte Carlo generator for l^+l^- production in $\bar{p}p$ interactions

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Abstract

A Monte Carlo event generator for the process $\bar{p}p \to l^+l^-$, with $l = e, \mu$ or τ, when both the proton target and the antiproton beam are unpolarized, is described. The input cross section is a leading order calculation with a massive lepton in the final state. The generator is a useful first tool to study lepton production at PANDA.

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Contents

1 Introduction

The availability of a high-intensity antiproton beam up to 15 GeV at the FAIR facility and of the PANDA detector offers unique possibilities for new investigations of the hadron structure (see [1] for a review). Feasibility studies for the determination of the proton electromagnetic form factors in the time-like region [2] with the PANDA detector have been already performed through the annihilation process $\bar{p}p \to l^+l^-$, with $l = e$ or μ, at several antiproton beam energies [3]. At the lowest order, the underlying mechanism is assumed to be the exchange of one virtual photon of four-momentum squared q^2. The difficulty of the measurement is related to the hadronic background, mostly annihilation into pions, which is estimated to be about six orders of magnitude larger than the production of the lepton pair. The development of efficient algorithms for signal reconstruction and background rejection requires the use of realistic Monte Carlo event generators for both lepton and pion production, so the differences in the distributions of the relevant kinematic variables can be used to tune the cuts needed to discriminate the signal from the background. In this note we describe in detail a Monte Carlo event generator for the process $\bar{p}p \to l^+l^-$, with $l = e, \mu$ or τ, when both the proton target and the antiproton beam are unpolarized. The cross section used by the generator is a leading order (LO) calculation where the lepton is massive.

2 Kinematics

Throughout this note, natural units $\hbar = c = 1$ are used. The components p^μ of a generic four-momentum p are labelled by the integer index μ, which takes the values $\mu = 0, 1, 2, 3$. The temporal componet p^0 refers to the energy E and the spatial components p^i, with $i = 1, 2, 3$, to the 3-dimensional part $\mathbf{p}$ of the four-momentum. Therefore, we write $p = (E, \mathbf{p})$. The Minkowski metric with signature $g = \mathrm{diag}(+, -, -, -)$ is used. With this choice, the square of the four-momentum becomes $p^2 = p^T g\, p = E^2 - \mathbf{p}^2$, where the superscript T stands for matrix transpose. On-shell particles therefore satisfy the relation $p^2 = E^2 - \mathbf{p}^2 = m^2$, where m is the mass of the particle.

The kinematic description of the process $\bar{p}(p_1)\; p(p_2) \to l^+(p_3)\; l^-(p_4)$, where $p_1,\; p_2,\; p_3$ and p_4 are the particle four-momenta, will be done in both the *laboratory frame*, or LAB frame for short, and the $\bar{p}p$ *center of mass frame*, or CM frame for short.

The LAB frame is a coordinate system defined by choosing the positive z-axes aligned with the antiproton beam momentum direction and having the proton target at rest. It is the frame in which a fixed target experiment is done and therefore, where the PANDA detector is at rest. Unprimed four-vectors will be used when we refer to the LAB frame. A picture of the kinematics in the LAB frame is shown in Fig 1.

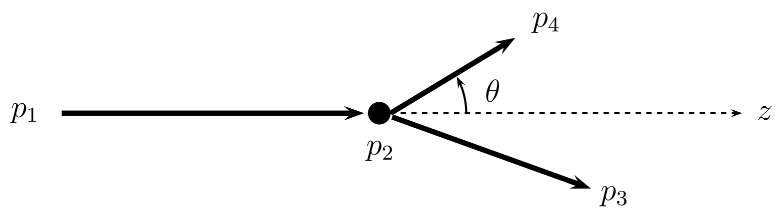

Figure 1: *Kinematics in the LAB frame.*

In the LAB frame, the initial state kinematics is completely specified by a single variable. For convenience, we take this variable to be the antiproton beam momentum P. The incoming proton and antiproton four-momenta are then given by:

$$p_1 = (E, 0, 0, P) , \qquad E = \sqrt{M^2 + P^2} \tag{2.1}$$

$$p_2 = (M, 0, 0, 0) , \tag{2.2}$$

where M is the mass of the proton.

The Lorentz invariant s, defined as as the invariant mass square of the proton-antiproton system, will be a useful quantity to solve the kinematics, as it corresponds to the available center of mass energy for particle production. Using initial state LAB frame variables, it can be easily evaluated:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ &= p_1^2 + p_2^2 + 2\, p_1 \cdot p_2 \\ &= M^2 + M^2 + 2\, EM \\ &= 2\, M(M + E) . \end{aligned} \tag{2.3}$$

The CM frame is the frame where the total momentum is zero, so we have $\mathbf{p}'_1 + \mathbf{p}'_2 = 0$ Primed four-vectors will be used when we refer to the CM frame. The incoming proton and antiproton collide, therefore, head to head. Again, we choose $\mathbf{p}'_1$ to define the $+z$ direction of the coordinate system.

The parametrization of the phase space of the outgoing leptons, that is, the set of all four-momenta p'_3 and p'_4 compatible with energy-momentum conservation, becomes simple in the CM frame. Momentum conservation means that we also have $\mathbf{p}'_3 + \mathbf{p}'_4 = 0$, so the two outgoing leptons are produced back to back, with opposite momenta, in the CM frame. A picture of the kinematics in the CM frame is shown in Fig 2.

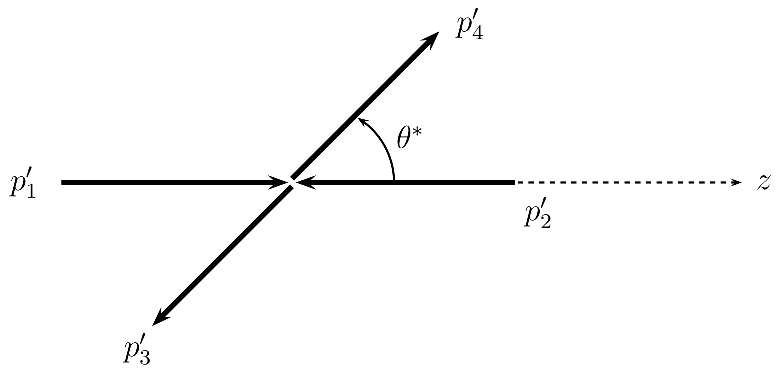

Figure 2: *Kinematics in the CM frame.*

Because for the two outgoing particles we have $\mathbf{p}'_3 + \mathbf{p}'_4 = 0$ and $m_3 = m_4 = m_l$, where m_l is the mass of the outgoing lepton, they are produced with the same energy, so $E'_3 = E'_4$. Energy conservation then means that each of them takes exactly half of the available energy in the CM frame:

$$E'_3 = E'_4 = \frac{\sqrt{s}}{2} \; , \tag{2.4}$$

which fixes the modulus of the momentum carried by each one of the leptons:

$$|\mathbf{p}'_3| = |\mathbf{p}'_4| = \sqrt{\frac{s}{4} - m_l^2} \; . \tag{2.5}$$

The only degrees of freedon that are still left to completely fix the kinematics of the final state are the polar and azimuthal angles in the CM frame θ^* and ϕ^* of one of the two leptons. For convenience, we choose the negative charged lepton as the reference. A more natural parametrization of the phase space is yielded by taking, instead of θ^*, $\cos\theta^*$ as an independent variable. $\cos\theta^*$ can take any value in the interval $[-1, 1]$ and ϕ^* and take any value in the interval $[0, 2\pi]$. The variables $\cos\theta^*$ and ϕ^* completely parametrize the phase space of the outgoing leptons in the CM frame:

$$p'_3 = (E'_4, -\mathbf{p}'_4) \; , \qquad \mathbf{p}'_4 = |\mathbf{p}'_4|(\sin\theta^* \cos\phi^*, \sin\theta^* \sin\phi^*, \cos\theta^*) \tag{2.6}$$

$$p'_4 = (E'_4, \mathbf{p}'_4) \tag{2.7}$$

The choice of the antiproton beam momentum direction to define z-axes in both the CM and LAB frames implies that the transition from one to the other is given by a Lorentz boost Λ along the momentum third component. The Lorentz boost then mixes the energy and

z-component of the a generic four-momentum k , while leaving the transverse components x and y unchanged. The explicit implementation of the non-trivial part of the boost is given by

$$\begin{pmatrix} k^0 \\ k_z \end{pmatrix} = \begin{pmatrix} \gamma & \sqrt{\gamma^2-1} \\ \sqrt{\gamma^2-1} & \gamma \end{pmatrix} \begin{pmatrix} k^{0\,\prime} \\ k_z' \end{pmatrix} , \qquad \gamma = \frac{\sqrt{s}}{2M} . \tag{2.8}$$

After boosting the two lepton final state to the LAB frame, the resulting momenta p_3 and p_4 constitute the final event record:

$$\begin{aligned} p_3' \to p_3 &= \Lambda p_3' \\ p_4' \to p_4 &= \Lambda p_4' . \end{aligned} \tag{2.9}$$

3 The cross section

The differential cross section for the annihilation process $\bar{p}(p_1)\ p(p_2) \to l^+(p_3)\ l^-(p_4)$ was first obtained in ref [4], and later on recalculated [5]. At the lowest order, the proccess occurs through the so called *one photon exchange* (OPE) mechanism, that is, the annihilation of the initial hadron state $\bar{p}p$ into a virtual photon γ^* which then decays to the lepton pair l^+l^- observed in the final state. The only diagram which contributes to the tree-level scattering amplitude is shown in Fig 3.

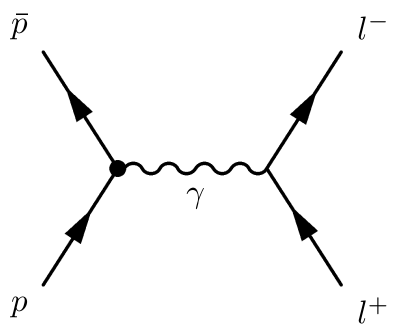

Figure 3: *LO contributing diagram to $\bar{p}p \to l^+l^-$*

Four-momentum conservation at the hadronic vertex implies that the four-momentum squared q^2 carried by the virtual photon is given, at the lowest order, by

$$q^2 = (p_1 + p_2)^2 = s . \tag{3.1}$$

At this order of approximation and keeping in the calculation the mass of the final state lepton m_l, the differential cross section for the process is given by

$$\frac{d\sigma}{d\cos\theta^*} = \frac{\pi\alpha^2}{2s}\,\frac{\beta_l}{\beta_p}\,\left\{\left(1+\frac{4m_l^2}{s}+\beta_l^2\cos^2\theta^*\right)|G_M|^2+\frac{1}{\tau}\left(1-\beta_l^2\cos^2\theta^*\right)|G_E|^2\right\} \quad , \tag{3.2}$$

where

$$\beta_l = \sqrt{1-4m_l^2/s} \tag{3.3}$$

$$\beta_p = \sqrt{1-4M^2/s} \tag{3.4}$$

$$\tau = \frac{q^2}{4M^2} \ , \tag{3.5}$$

and where $|G_E|$ and $|G_M|$ are the modulus of the (Sachs) electric and magnetic proton form factors, respectively. The threshold $s > 4M^2 > 4m_{e,\mu}^2$ ensures that electron and muon production is always possible at any value of proton-antiproton center of mass energy. Tau production starts only at $s > 4m_\tau^2$. In the case of electron production, terms of the order $(m_e/M)^2 \sim 10^{-7}$ can be safety neglected by formally setting $m_l = 0$ (which implies $\beta_l = 1$) in Eq. 3.2, which then becomes

$$\frac{d\sigma}{d\cos\theta^*} = \frac{\pi\alpha^2}{2s}\,\frac{1}{\beta_p}\,\left\{\left(1+\cos^2\theta^*\right)|G_M|^2+\frac{1}{\tau}(1-\cos^2\theta^*)|G_E|^2\right\} \quad . \tag{3.6}$$

In this event generator, only the full cross section given by Eq. 3.2 is used for all e, μ and τ production, even when Eq. 3.6 could be taken as an accurated approximation in the case of electron production.

With a little bit of algebra, it is strightforward to see that the cross section 3.2 can be rewritten in the form

$$\frac{d\sigma}{d\cos\theta^*} = N(1+A\cos^2\theta^*) \ , \tag{3.7}$$

where the factors N and A are given by

$$N = (2-\beta_l^2)|G_M|^2+\frac{1}{\tau}|G_E|^2 \tag{3.8}$$

$$A = \frac{\beta_l^2(1-R^2/\tau)}{2-\beta_l^2+R^2/\tau} \ , \qquad R \equiv \frac{|G_E|^2}{|G_M|^2} \quad . \tag{3.9}$$

Observing that $\beta_l^2 < 1$, we can drop the positive constant (i.e. not $\cos\theta^*$ dependent) N in Eq. 3.7 and use

$$f(\cos\theta^*) \equiv 1 + A\cos^2\theta^* \tag{3.10}$$

as the (non-normalized) probability density function for event generation purposes only. An upper bound C to the function f in the full kinematic region $-1 < \cos\theta^* < 1$ is given by

$$f < C \; , \qquad C = 1 + |A| \; . \tag{3.11}$$

4 Sampling the cross section

The unnormalized probability density f given by Eq. 3.10 as a function of $\cos\theta^*$ was the starting point for event generation. Event generatation proceeds with the simple "accept/reject" method. In the event loop stage, the probability density f is repeatedly sampled and tested as follows until the desired number of events is reached: 1) a value for $\cos\theta^*$ is generated randomly in the user defined interval $[\cos\theta^*_{min}, \cos\theta^*_{max}]$ with flat probability density, and the value is used to evaluate the function f; 2) a second random variable y is generated uniformly in the range $[0, C]$, where C is the upper bound to f in the full kinematic range as given by Eq. 3.11; 3) if y is larger than the value of f, the event is discarded and the function f resampled (step 1); otherwise 4) the event is accepted, a value ϕ^* is uniformly generated in the range $[0, 2\pi]$ and 5) the event is built by calculating the two lepton final state four-momenta out of the generated $\cos\theta^*$ and ϕ^* according to the description given in Section 2.

The pseudo random number generator used to produced uniformly distributed random numbers in the interval $[0, 1)$ was RANLUX [6], whose source code [7] was added to our program. Typical generation times were found to be of the order of 5 μ sec per accepted event.

5 Routines in the generator

A brief description of all routines used by the generator is provided here.

- `mz_pp_to_leplep_vandewi_init`: it checks that all user defined parameters have their values in the allowed ranges, otherwise it kills the job; it initialises the random number generator and prints out to the log file basic kinematic information.

- `mz_pp_to_leplep_vandewi_sigma_nonorm`: it returns the value of the unnormalized cross section f given by Eq. 3.10 corresponding to the final state lepton and input antiproton momentum at the generated $\cos\theta^*$.

- `mz_pp_to_leplep_vandewi_event`: it generates one physical event by returning the four-momenta of the two charged leptons in the final state, in the LAB frame.

- `mzvmod2`: it returns the square of an input four-vector.

- `mzrnd`: it returns a random number generated with flat probability density in the in the interval $[a, b]$ using RANLUX.

- `mzboost`: it boosts the four momenta of the two leptons in the final state from the CM frame to the LAB frame.

6 Interface to PandaRoot

The $\bar{p}p \to l^+l^-$ event generator has been successfully interfaced to PandaRoot, so the output is directly streamed to the simulation framework during execution time, event by event. All source files can be found in `/pandaroot/pgenerators/EMFFgenerators`. The files PndLepLepGenerator.* contain the interface class, which is responsible of the communication between the original event generator source code and PandaRoot. The class receives particle momenta from the event generator and pass this information efficiently to PandaRoot, which then procceds with the simulation stage. Sucessful compilation of the generator within the simulation framework was achieved by aplying the appropiate modifications to the CMakeLists.txt files.

7 User guide

The set of parameters needed by the generator are suplied and passed to the program in the simulation macro, where the generator is called. The user has to provide the following data:

- *seed*: the initialisation seed fixing the state in RANLUX, an integer in the range $[1, 2^{31})$.

- *lepton flag*: flag to set the final state lepton. The possible values are 0 (electron), 1 (muon) and 2 (tau).

- P: antiproton beam momentum in the lab frame, in GeV. The allowed range is $P \geq 0$ GeV. Setting a value below the PANDA threshold $P < 1.5$ GeV or above the upper limit $P > 15$ GeV is possible, but a warning message will be printed out to the log file. Tau production ($lepton\ flag = 2$) is only possible above the threshold $s > 4m_\tau^2$, which corresponds to antiproton beam momentum $P > 5.71$ GeV.

- $|G_E|/|G_M|$: ratio of the (Sachs) electric and magnetic proton form factors at $q^2 = s$. The allowed range is $|G_E|/|G_M| \geq 0$. As at the lowest order $q^2 = s$, and the center of mass energy squared s is fixed by the initial state kinematics, q^2 is a constant for all generated events, so no dynamical models of the form factors are really needed.

- $\cos\theta^*_{min}$ and $\cos\theta^*_{max}$: minimum and maximum values, respectively, of $\cos\theta^*$. The allowed range is $-1 \leq \cos\theta^*_{min} < \cos\theta^*_{max} \leq 1$.

As already mentioned, the program checks that all user defined parameters take their value in the allowed ranges, killing the job when this condition is not satisfied. An example simulation macro is given in Appendix A

8 Examples

The generator has been examined carefully in order to check that the resulting distributions at the Monte Carlo true level agree with the expectations. Figs. 4, 5 and 6 show example distributions of $\cos\theta^*$ for e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$ generated events with antiproton beam momentum $P = 4.0$ GeV (electron, muon) and $P = 8.0$ GeV (tau), under the three different hipotheses $|G_E|/|G_M| = 0, 1$ and 3. In all cases, the resulting distribution follows the corresponding cross section 3.2, or, equivanlently, the unnormalized probability density function 3.10. The kinematics of the two body decay in the $\bar{p}p$ CM frame becomes manifest in Fig. 7, where the production of the two outgoing leptons (e^+e^-) in a "back to back" configuration with the expected energy $\sqrt{s}/2$ and flat azymuthal distribution is clearly shown. The distribution of a few variables after boosting the events to the LAB frame is shown Fig. 8. The energy spectrum of the electron in the LAB frame mimics the distribution of $\cos\theta^*$ in the CM frame, as corresponds to the linear mapping between these two variables induced by the boost along the momentum third component. The azymuthal flat distribution is preserved by the boost, as expected, whereas the whole sample is boosted in the forward direction, as the $\cos\theta$ distribution shows.

9 Summary

A Monte Carlo event generator for the process $\bar{p}p \to l^+l^-$, with $l = e, \mu$ or τ, when both the proton target and the antiproton beam are unpolarized, has been described. The input cross section is a leading order calculation with a massive lepton in the final state. The generator is a useful first tool to study lepton production at PANDA.

Acknowledgements

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A An example simulation macro

For the convenience of the user, we provide in this appendix the example simulation macro `Simulation_Example.C` which calls the lepton generator. The reader is assumed to have some familiarity in doing simulations with PANDA ROOT, so only the relevant parts related to the Monte Carlo generator are explicitly shown. The parameters needed by the generator, described in Section 7, are passed through the arguments of the function `Simulation_Example` in the macro.

```
Simulation_Example( Int_t seed=1,
                    Int_t lepton_flag=1,
                    Double_t P=5.0,
                    Double_t R=3.0,
                    Double_t cos_theta_min = -0.8,
                    Double_t cos_theta_max = 0.8 ){

   /**
   *    PARAMETERS:
   *
   *      seed:             RANLUX seed
   *      lepton_flag:      final state lepton identifier
   *      P:                pbar momentum LAB frame [GeV]
   *      R:                |G_E|/|G_M| ratio
   *      cos_theta_min:    minimum cos(theta*)
   *      cos_theta_max:    maximum cos(theta*)
   *
   *    Ranges:
   *
   *      1 <= seed < 2^31
   *      lepton_flag = 0, 1, 2      (0=electron, 1=muon, 2=tau)
   *      P >= 0.0 GeV
   *      R >= 0.0
   *      -1 <= cos_theta_min < cos_theta_max <= 1
   */
```

```
                    ...

// Load basic libraries
gROOT->LoadMacro("$VMCWORKDIR/gconfig/rootlogon.C");
rootlogon();
gSystem->Load("libEMFFGEN");

                    ...



// Create and Set Event Generator
//-------------------------------
FairPrimaryGenerator* primGen = new FairPrimaryGenerator();
fRun->SetGenerator(primGen);

PndLepLepGenerator* leplepGen = new PndLepLepGenerator();
leplepGen->SetSeed(seed);
leplepGen->SetParticleID(lepton_flag);
leplepGen->SetBeamMom(P);
leplepGen->SetGeGmRatio(R);
leplepGen->SetCosThetaMin(cos_theta_min);
leplepGen->SetCosThetaMax(cos_theta_max);
primGen->AddGenerator(leplepGen);

                    ...

exit(0);
}
```

References

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[7] available online at the author web page:
`http://luscher.web.cern.ch/luscher/ranlux/index.html`.

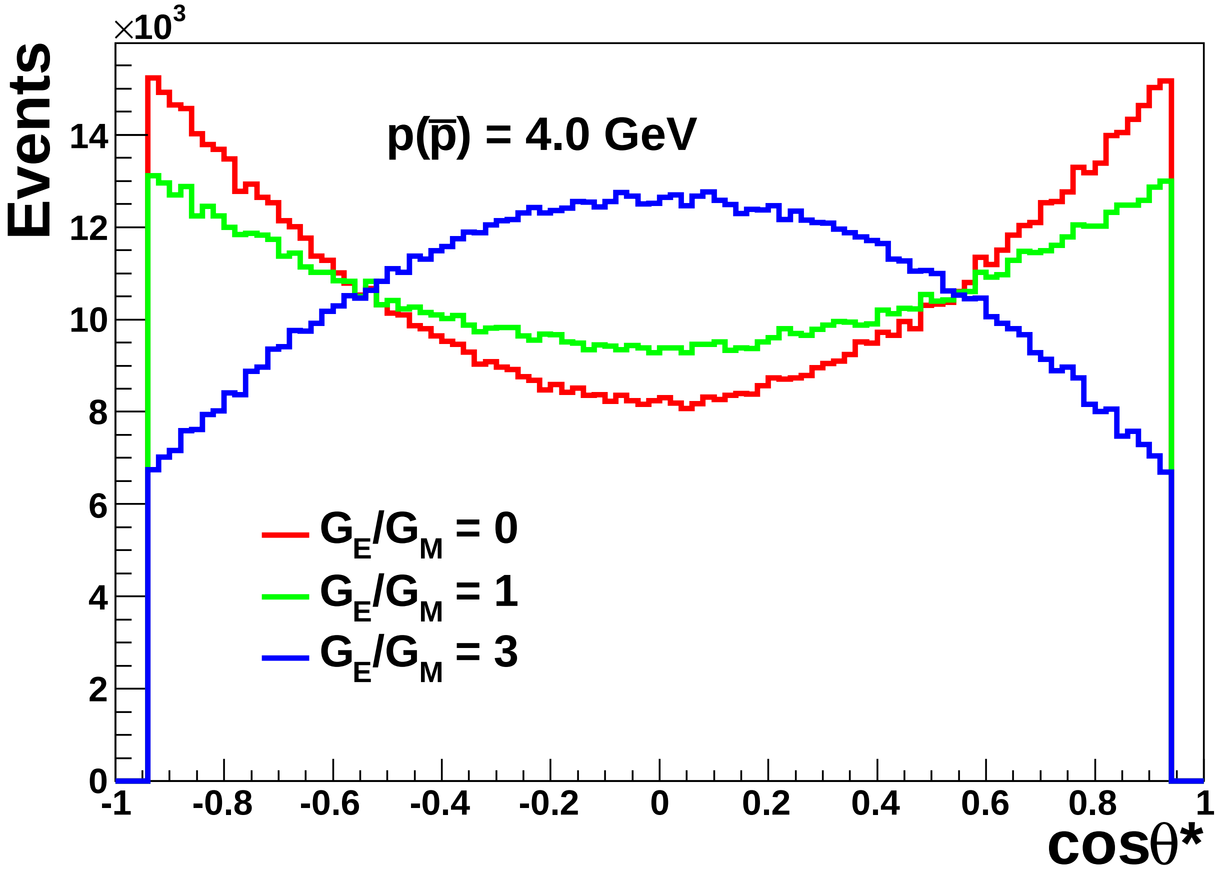

Figure 4: *Distribution of* $\cos\theta^*$ *in a sample of* 10^6 $\bar{p}p \rightarrow e^+e^-$ *generated events for antiproton beam momentum in the lab frame* 4.0 *GeV, in the range* $-0.94 < \cos\theta^* < 0.94$*, under the hipothesis* $|G_E|/|G_M| = 0$ *(red),* $|G_E|/|G_M| = 1$ *(green) and* $|G_E|/|G_M| = 3$ *(blue).*

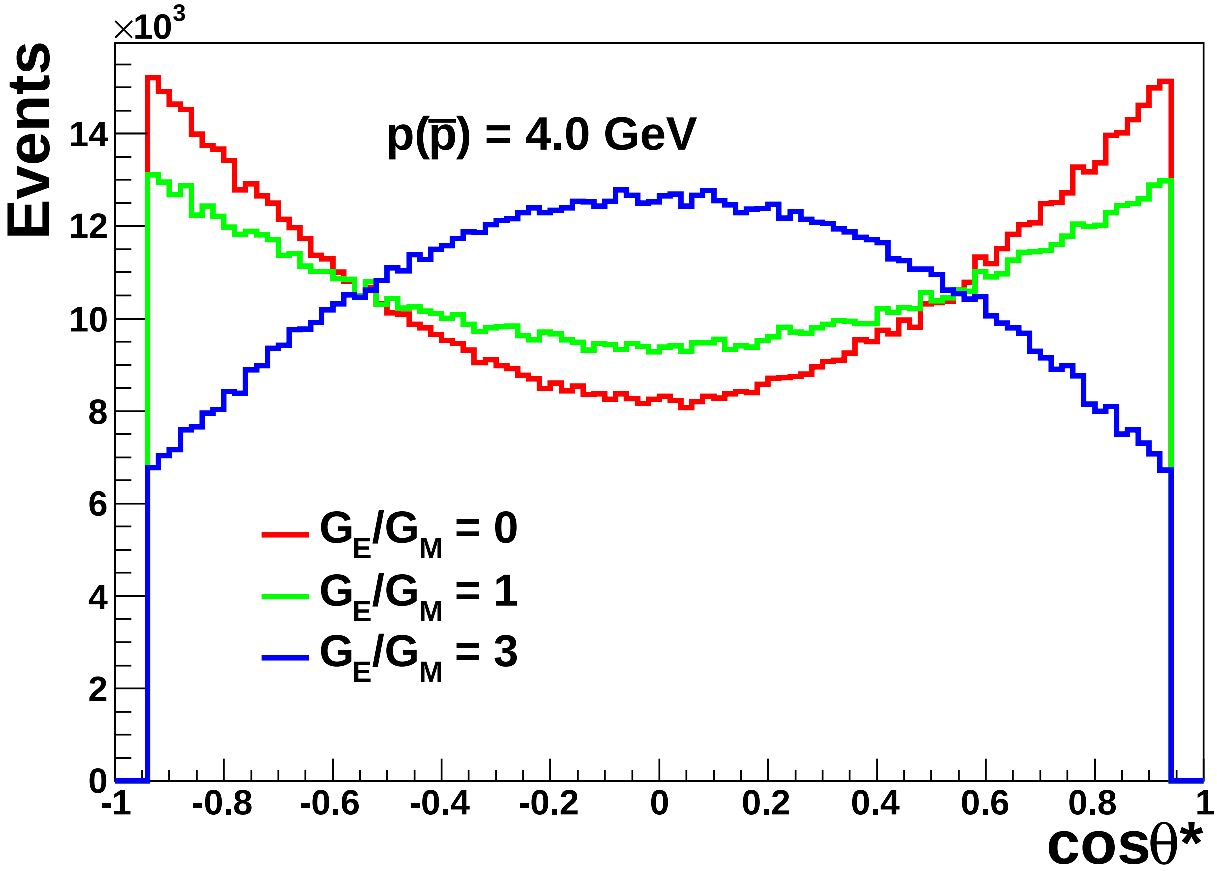

Figure 5: *Distribution of* $\cos\theta^*$ *in a sample of* 10^6 $\bar{p}p \to \mu^+\mu^-$ *generated events for antiproton beam momentum in the lab frame* 4.0 *GeV, in the range* $-0.94 < \cos\theta^* < 0.94$*, under the hipothesis* $|G_E|/|G_M| = 0$ *(red),* $|G_E|/|G_M| = 1$ *(green) and* $|G_E|/|G_M| = 3$ *(blue).*

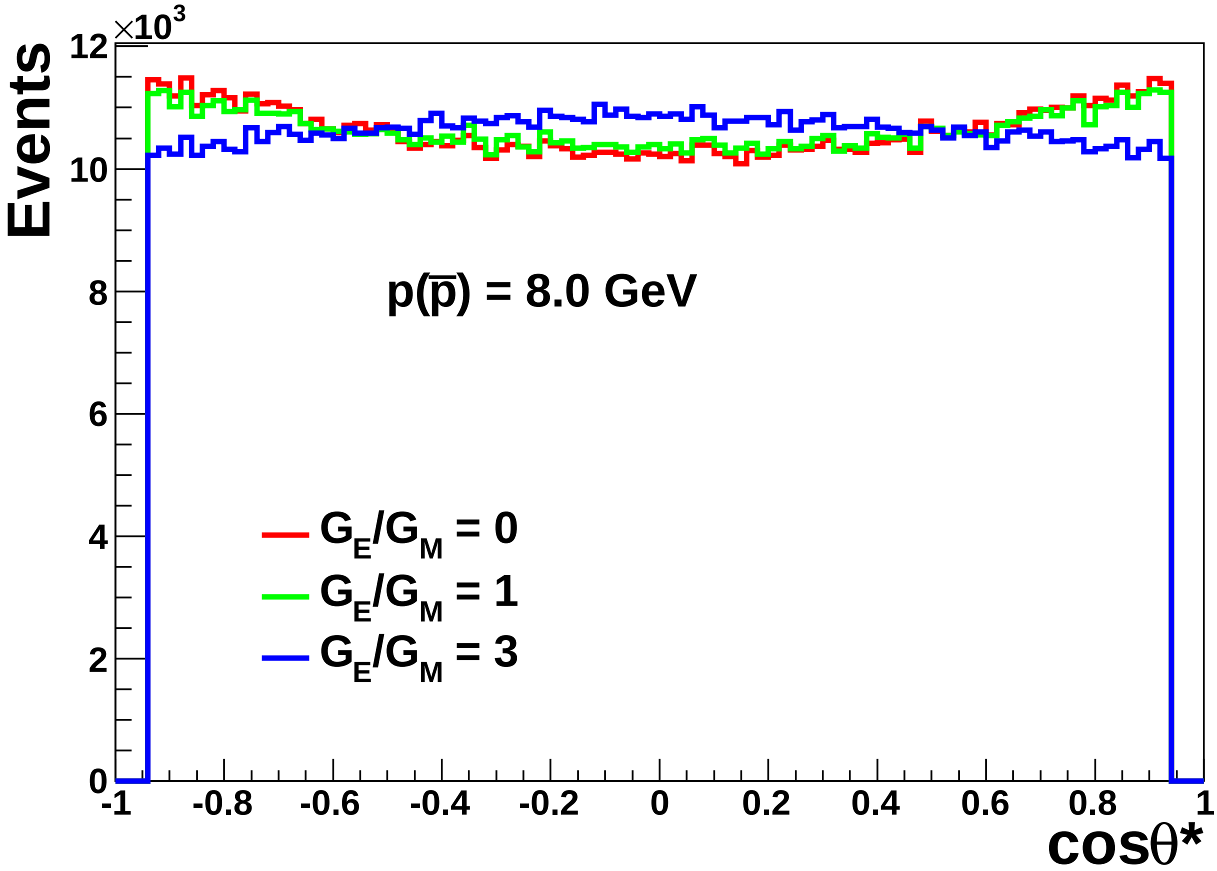

Figure 6: *Distribution of* $\cos\theta^*$ *in a sample of* 10^6 $\bar{p}p \to \tau^+\tau^-$ *generated events for antiproton beam momentum in the lab frame* 8.0 *GeV, in the range* $-0.94 < \cos\theta^* < 0.94$*, under the hipothesis* $|G_E|/|G_M| = 0$ *(red),* $|G_E|/|G_M| = 1$ *(green) and* $|G_E|/|G_M| = 3$ *(blue).*

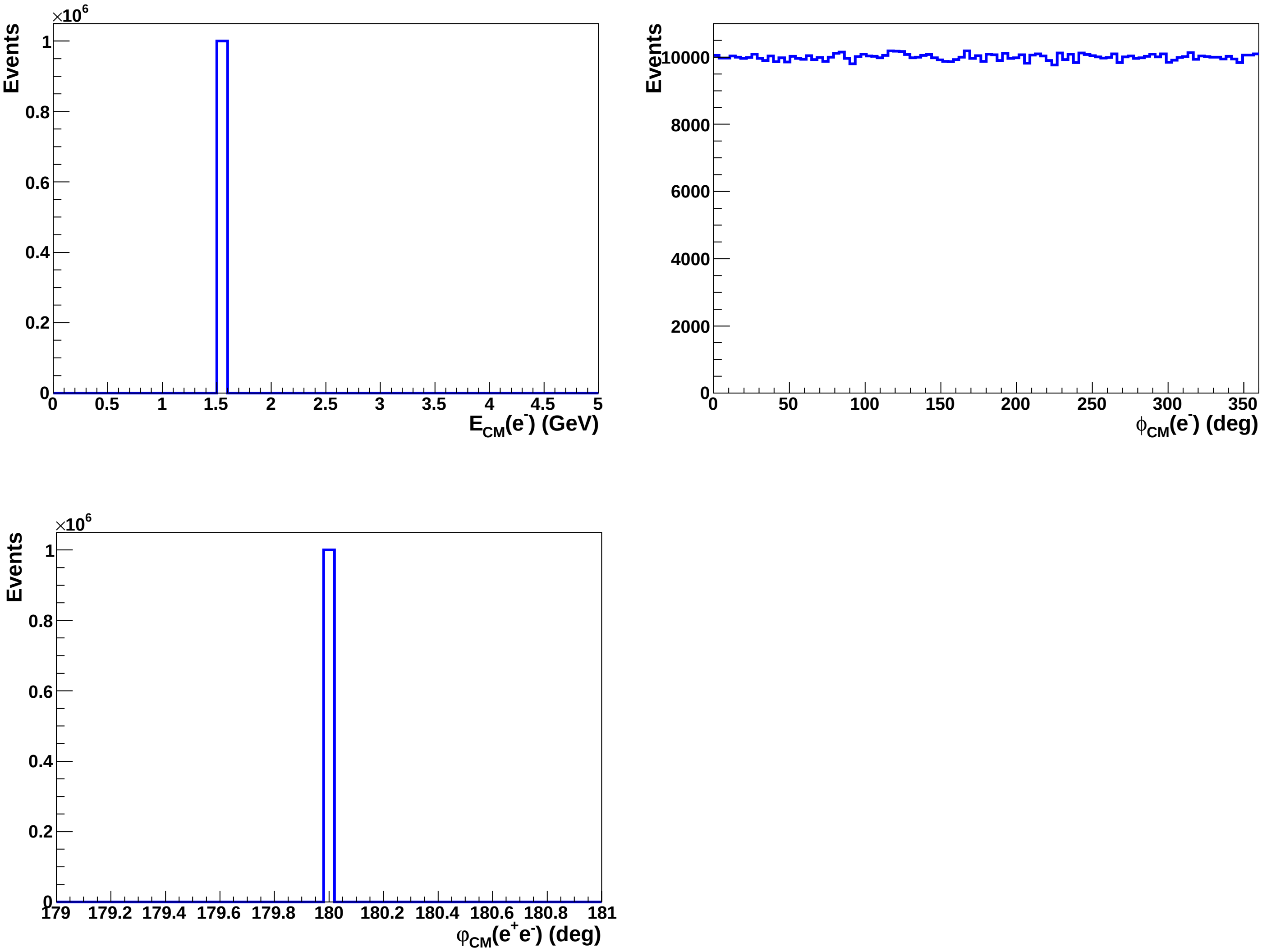

Figure 7: *The distribution of the energy and the azymuthal angle of the electron (top) and the angle between the electron and the positron (down) in the $\bar{p}p$ CM frame, in a sample of* 10^6 $\bar{p}p \to e^+e^-$ *generated events for antiproton beam momentum* $4.0\ GeV$*, under the hipothesis* $|G_E|/|G_M| = 0$.

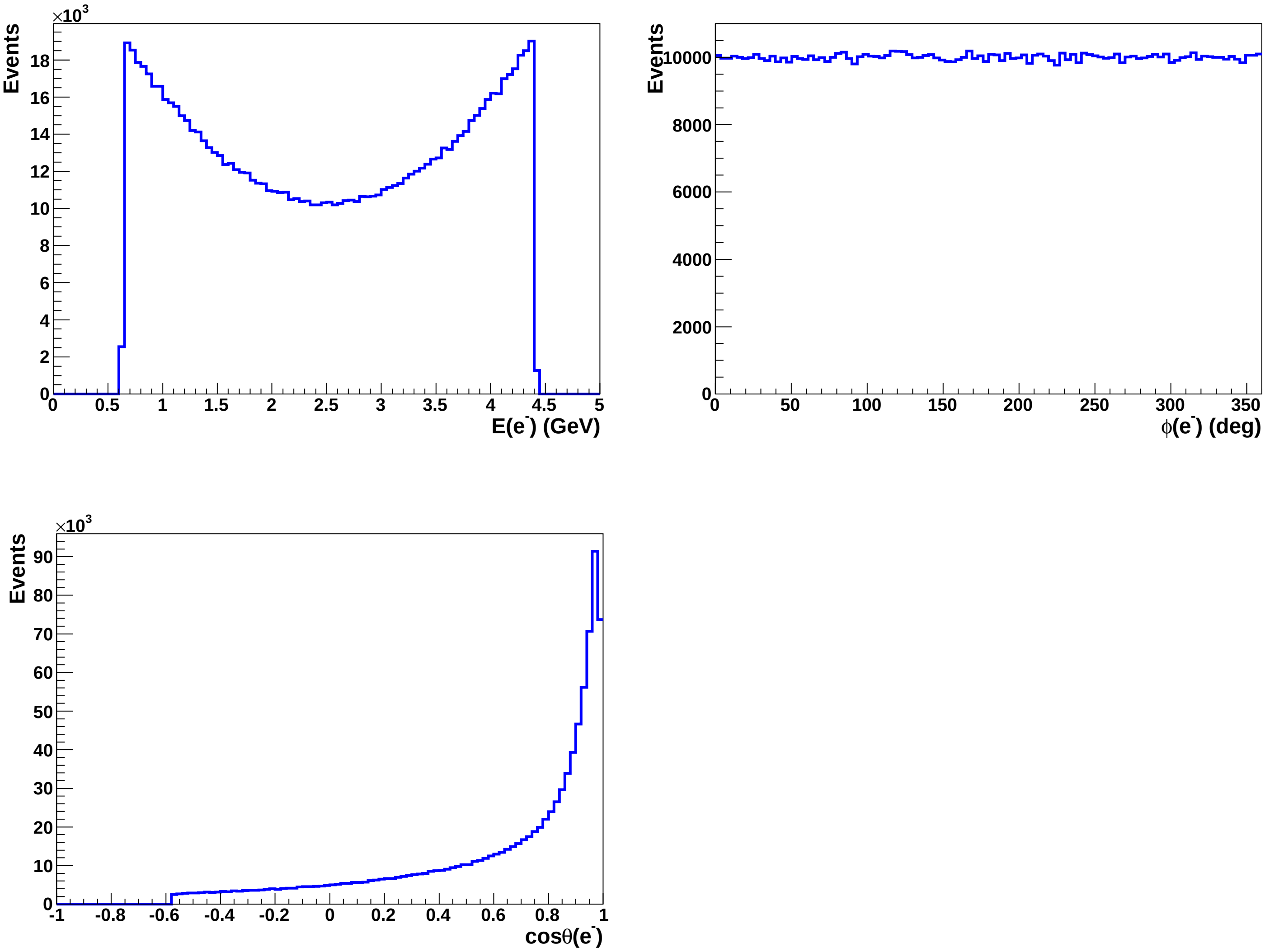

Figure 8: *The distribution of the energy and the azymuthal angle (top) and* $\cos\theta$ *(down) of the electron in the LAB frame, in a sample of* 10^6 $\bar{p}p \to e^+e^-$ *generated events for antiproton beam momentum* 4.0 *GeV, under the hipothesis* $|G_E|/|G_M| = 0$.