# Time-Like Form Factors of the Neutron @ BESIII

## Vortrag im Arbeitsgruppenseminar

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21. Juni 2012

# Outline

# Motivation and Goals

Motivation:

- Time-like form factors of the nucleon poorly measured
- For neutron only 1(!) serious measurement (FENICE) with very poor statistics (74 events)
- pQCD prediction $|G_M^n| = \frac{1}{2}|G_M^p|$, but from data $|G_M^n| = \frac{3}{2}|G_M^p| \rightarrow$ pysical explanation?
- Babar measured unexpected behavior of the proton form factors at threshold $\rightarrow$ behavior at threshold for neutron from factor?

Goals:

- Extraction of the cross section for $e^+e^- \rightarrow n\bar{n}$
- Extraction of $|G_M^n|$ and $|G_E^n|$, at least the ratio $\mathrm{R} = |G_M^n|/|G_E^n|$
- Investigation of the behavior at threshold

# Theoretical Basics

1. Form Factors
2. Space-Like Region
3. Time-Like Region

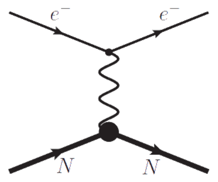
# Electromagnetic Form Factors of the Nucleon

- Electromagnetic form factors are the most direct link to the structure of the nucleon in therms of its constituents
- Describe the coupling of a photon to the distribution of charges and currents in the nucleon (Born approximation)
- Can be real (space-like) or complex (time-like) functions

### Space-Like ($q^2 < 0$)

Elastic eN-Scattering:

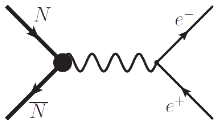
$$e^- + N \rightarrow e^- + N$$



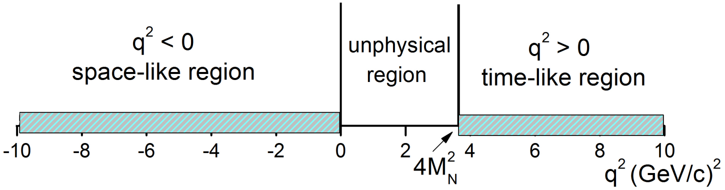

### Time-Like ($q^2 > 0$)

Annihilation:

$$e^- + e^+ \rightarrow N + \overline{N}$$



# Electromagnetic Form Factors of the Nucleon




**Space-Like ($q^2 < 0$)**

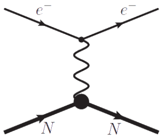
- Real
- Rosenbluth cross section
- No single spin observables
- Double spin observables

**Time-Like ($q^2 > 0$)**

- Complex, imaginary part: polarisation
- Rosenbluth cross section
- Single spin observables
- Double spin observables

## Space-Like Region

- Elastic scattering: $e^- + \mathrm{N} \to e^- + \mathrm{N}$:




- **Vector current:**

$$\langle N(p_2)|J^{\mu}_{em}|N(p_1)\rangle = \overline{U}(p_2)\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}}{2M}q_{\nu}\right]U(p_1)$$

- **Dirac & Pauli form factors $F_1$, $F_2$:**

$$F_{1,p}(0) = 1; \qquad F_{2,p}(0) = \mu_p; \qquad F_{1,n}(0) = 0; \qquad F_{2,n}(0) = \mu_n$$

- **Sachs form factors:**

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2) \qquad G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$

# Space-Like Region

Approaches:

## Rosenbluth cross section:

Obtain form factors of unpolarised cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \frac{\tau}{\epsilon(1+\tau)} \left[G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2)\right]$$

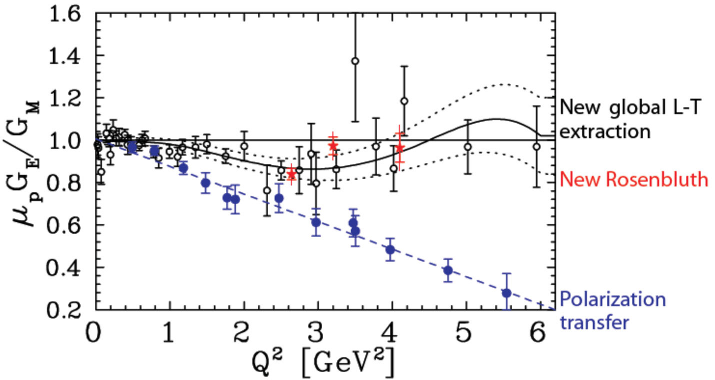
$$Q^2 = -q^2 \qquad \tau = Q^2/4M_N^2 \qquad \frac{1}{\epsilon} = 1 + 2(1+\tau)\tan^2\left(\frac{\theta}{2}\right)$$

## Polarisation transfer:

Measure outgoing nucleon polarisation

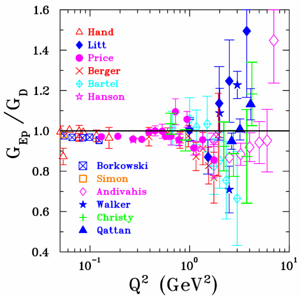
$$\frac{G_E(Q^2)}{G_M(Q^2)} = -\frac{E+E'}{2M_N}\tan^2\left(\frac{\theta^*}{2}\right)\frac{P_t}{P_l}$$

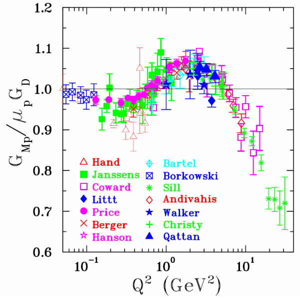
# Space-Like Electromagnetic Form Factors

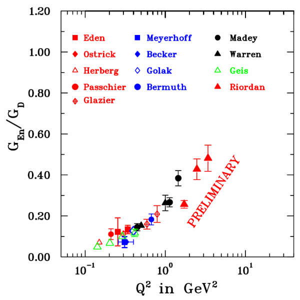


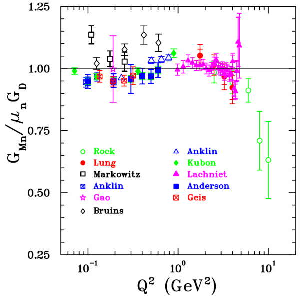

- High statistics for space-like region
- Discrepancy between different methods could be explained with two-photon exchange and radiative corrections

# Space-Like Electromagnetic Form Factors



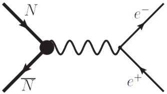



# Time-Like Region

- Annihilation $e^+ + e^- \rightarrow \mathrm{N} + \overline{\mathrm{N}}$:
  $\mathrm{p} + \overline{\mathrm{p}} \rightarrow e^+ + e^-$:




- **Vector current $\langle N(p_2)|J^{\mu}_{em}|N(p_1)\rangle \rightarrow$ crossing symmetry:**

$$\langle 0|J^{\mu}_{em}|N(p_1)\overline{N}(p_2)\rangle = \overline{U}(p_2)\left[F_1(q^2)\gamma^{\mu} - F_2(q^2)\frac{i\sigma^{\mu\nu}}{2M}q_{\nu}\right]U(p_1)$$

$$G_E(q^2) = F_1(q^2) + F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + \tau F_2(q^2)$$

- **Differential cross section:**

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2\beta C}{aq^2}\left[\frac{1}{\tau}G_E^2(q^2)\sin^2(\theta_N^{cm}) + G_M^2(q^2)(1+\cos^2(\theta_N^{cm}))\right]$$

$$\beta = \sqrt{1 - 4m_n^2/m^2} \qquad C = \frac{y}{(1-e^{-y})} \qquad y = \frac{\pi\alpha m_n}{\beta m}$$

# Time-Like Region - Characteristics

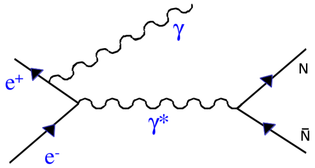
- Form factors are complex functions of $q^2$
  - determination of relative phase by measuring polarisation of outgoing N
- $G_E(4M_N^2) = G_M(4M_N^2)$ (at threshold), if $F_1$ and $F_2$ analytic
- $G_E^n(q^2)$ negligible at large $q^2$
- At large $q^2 \to G(q^2) = G(-q^2)$ (QCD, analycity)
- According to pQCD simplest prediction:

$$\left|\frac{G_M^n}{G_M^p}\right|^2 \approx \left(\frac{q_d}{q_u}\right)^2 = \frac{1}{4}$$

  - In models representing the nucleon in terms of valence quarks it should be $G_M^n > G_M^p$

# Time-Like Electromagnetic Form Factors

- Another approach:
  obtain form factors using ISR technique




## Initial State Radiation (ISR) method

- $\sigma(e^+e^- \to N\overline{N})$ can be measured from threshold ($q^2 = 4m_N^2$) to full cm-energy $\sqrt{s}$ by studying the ISR process $\sigma(e^+e^- \to N\overline{N} + \gamma)$

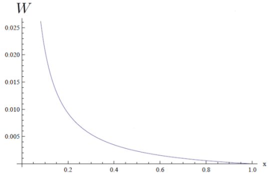
$$\frac{d^2\sigma_{e^+e^- \to N\overline{N}+\gamma}(m)}{dm\ dcos\theta_\gamma^*} = \frac{2m}{s} W(s, x, \theta_\gamma^*)\sigma_{N\overline{N}}(m)$$

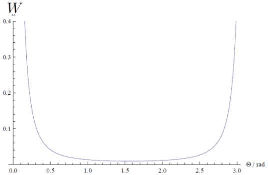
- with: $m = N\overline{N}$ invariant mass, $x = \frac{2E_\gamma^*}{\sqrt{s}} = 1 - \frac{m^2}{s}$, $E_\gamma^*$ and $\theta_\gamma^*$ are the ISR photon energy and polar angle in $e^+e^-$ cm frame

# Time-Like Electromagnetic Form Factors

- W is the probability of ISR photon emission for $\theta^*_\gamma >> \frac{m_e}{\sqrt{s}}$:

$$W(s, x, \theta^*_\gamma) = \frac{\alpha}{\pi x}\left(\frac{2 - 2x + x^2}{\sin^2(\theta^*_\gamma)} - \frac{x^2}{2}\right)$$




- The cross section for $e^+e^- \rightarrow N\overline{N}$ process is given by:

$$\sigma_{N\overline{N}}(m) = \frac{4\pi\alpha^2\beta C}{3m^2}\left[\frac{2m_N^2}{m^2}G_E^2(m) + G_M^2(m)\right]$$

- Two possibilities using ISR:
  - Tagged analysis: ISR photon detected, information about photon can be used for kinematic cuts
  - Untagged analysis: ISR photon not detected, angular distribution of ISR photon can be used for background suppression

# Time-Like Electromagnetic Form Factors

## 2$\gamma$-exchange in timelike region:

- Matrix element beyond Born approximation:

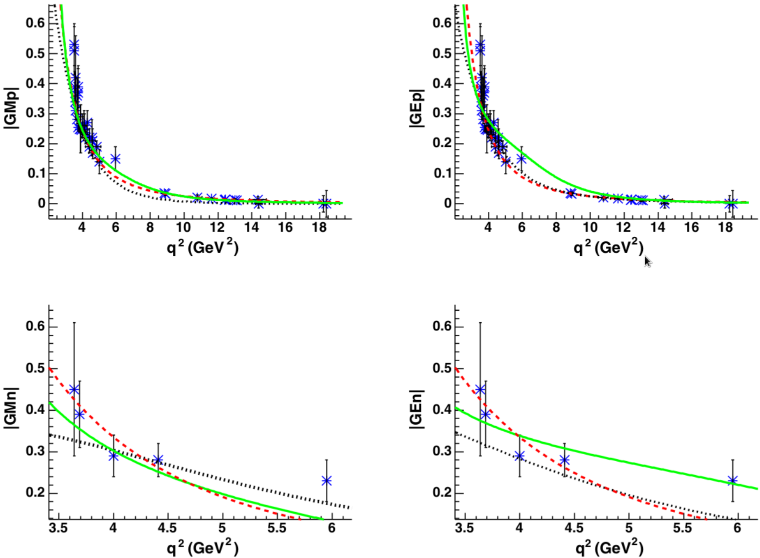
$$M = \frac{-ie^2}{q^2}\overline{u}(k_1)\gamma_\mu v(k_2)\ x\ \overline{v}(p_2)\left[\tilde{G}_M\gamma_\mu - \tilde{F}_2\frac{P^\mu}{m} + \tilde{F}_3\frac{P^\mu}{m^2}\not{K}\right]u(p_1)$$

- Cross section with 2$\gamma$-exchange:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2\beta C}{aq^2}\left[\frac{1}{\tau}G_E^2(q^2)\sin^2(\theta^*) + G_M^2(q^2)(1+\cos^2(\theta^*))\right.$$
$$+2\Re\left[G_M\delta\tilde{G}_M\right]\left(1+\cos^2(\theta^*)\right) + \frac{2}{\tau}\Re\left[G_E\delta\tilde{G}_M\right]\sin^2(\theta^*)$$
$$\left.+2\left[\Re\left[G_M\tilde{F}_3^*\right] - \frac{1}{\tau}\Re\left[G_E\tilde{F}_3^*\right]\right]\sqrt{\tau(\tau-1)}\cos(\theta^*)\sin^2(\theta^*)\right]$$

- 2$\gamma$-contribution can be measured by analysing the asymmetry in angular distribution in experiments with superior statistics, $\delta_{2\gamma} < 1\%$

# Time-Like Electromagnetic Form Factors




Results and predictions for time-like form factors. Dashed line: pQCD-inspired, dotted line: Iachello's parametrisation, solid line: Lomon Model

# Experimental Designs

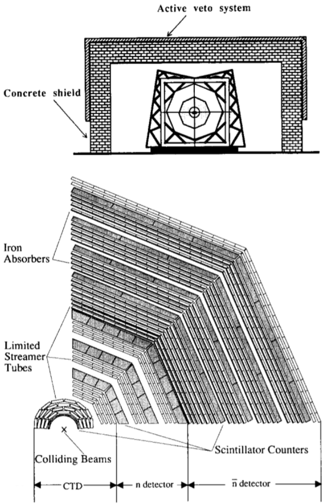
1. FENICE
2. BESIII

# FENICE - ADONE

- 1. measurement of the time-like form factors of the neutron (1991-1993)
- Prozess: $e^+e^- \to n\bar{n}$, $E_{cm} = (1.88,\ 1.90 - 2.44,\ 3.10)$ GeV

## FENICE-Detector:

- Limited streamer tubes as tracking devices
- Scintillation counters as timing and triggering devices
- Thin iron plates as distributed converters where $\bar{n}$ annihilate
- $E_{cm}$: $1.5 - 3.1$ GeV
- Luminosity: $1.2{\cdot}10^{29}\text{cm}^{-2}\text{s}^{-1}$ (threshold)
- Luminosity: $0.6{\cdot}10^{29}\text{cm}^{-2}\text{s}^{-1}$ (3.1 GeV)




FENICE-detector

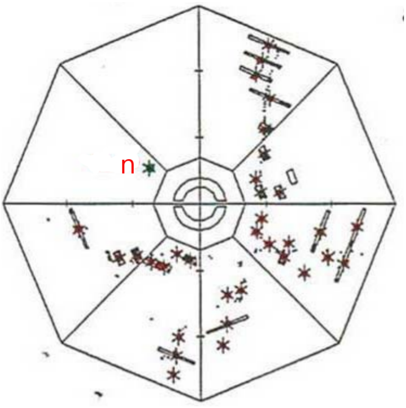
# FENICE - Event-Selection

## Star-topology:

- Many charged tracks, mainly pions, pointing to the $\overline{n}$ annihilation vertex
- Electromagnetic showers ($\pi^0$) pointing to the same vertex

## Event-selection:

- Identification of $\overline{n}$ by star-topology
- No indentification of $n$ (15% efficiency)
- Consistency check by looking at TOF
- No other charged/neutral particles in event




Example of a $n\overline{n}$ event

# FENICE - Results

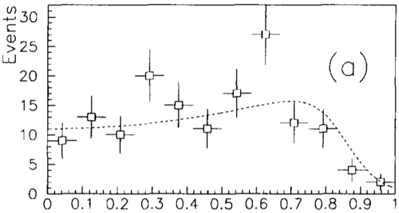
## Two cases:

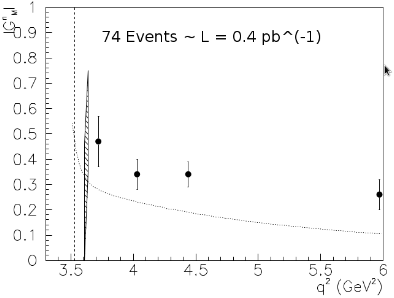
- $|G_E| = |G_M|$: $\rightarrow$ flat angular distributuion
- $|G_E| = 0$: $\rightarrow$ $(1 + \cos^2)$ angular distributuion

## Fit to data:

$$A \cdot (1 + \cos^2 \theta) + B \cdot \sin^2 \theta$$

- $A$ and $B$ are free parameters, related to $|G_M|$ and $|G_E|$
- Best fit obtained with case 2
- Shaded area $\rightarrow$ both cases



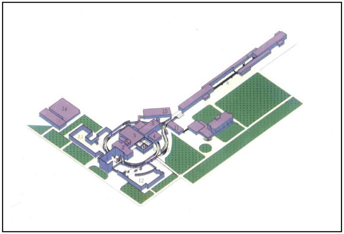

Results for $G_M^n$

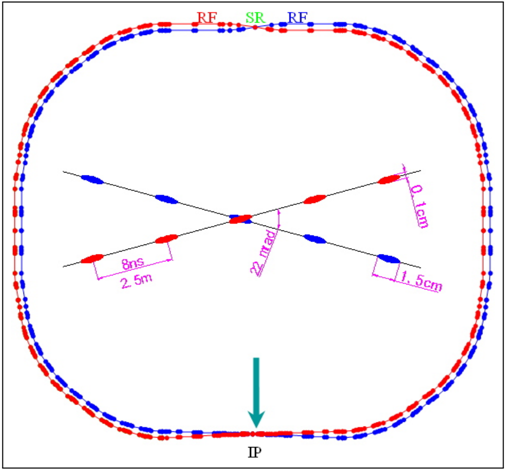
# BEPCII - Beijing

The Beijing Electron-Positron Collider II (BEPCII)

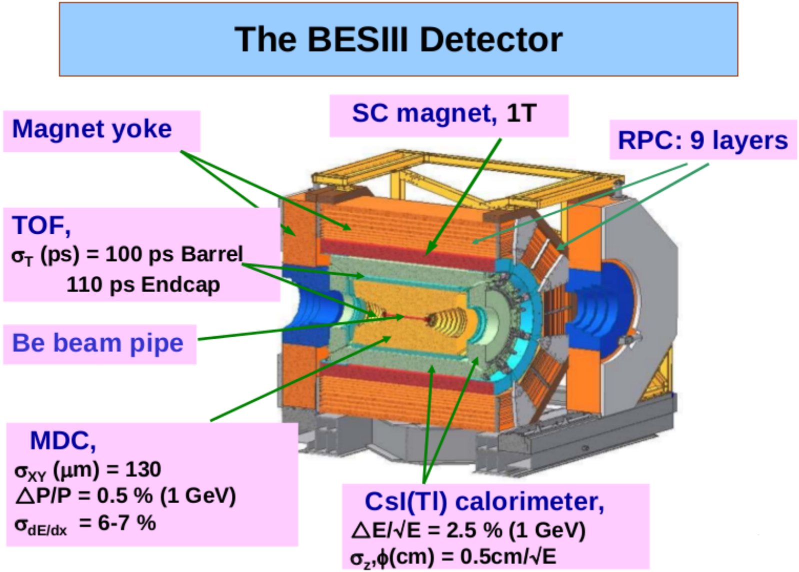
# BEPCII - Beijing

- Circumference: 237 m
- Beam energy: 1.0-2.3 GeV
- Luminosity: $10^{33}$ $cm^{-1}s^{-1}$
- $E_{cm}$: 3.770 GeV
- Energy spread: $5.16 \cdot 10^{-4}$
- Current: 0.91 A



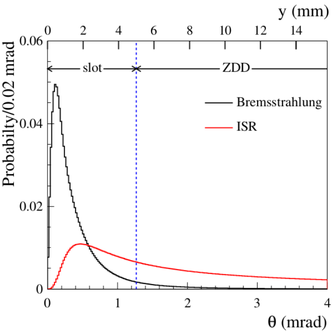


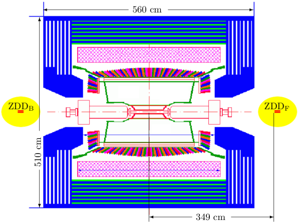
# BESIII-Detector

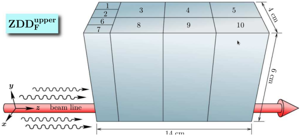


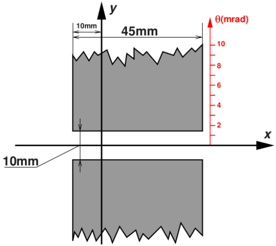
# New Zero-Degree-Detector

- ZDD: small calorimeter (scintillator fibres)
- angular distribution of ISR-$\gamma$ peaks at small angles
- without ZDD only 20% of ISR-$\gamma$ hit main detector
- new ZDD at least doubles acceptance



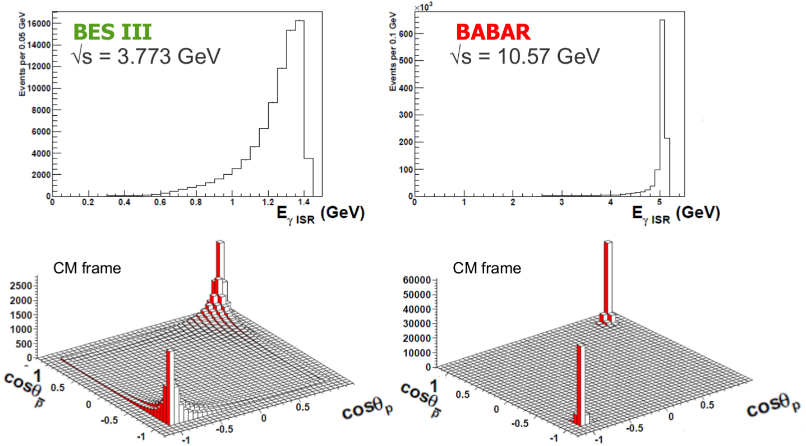



# Analysis

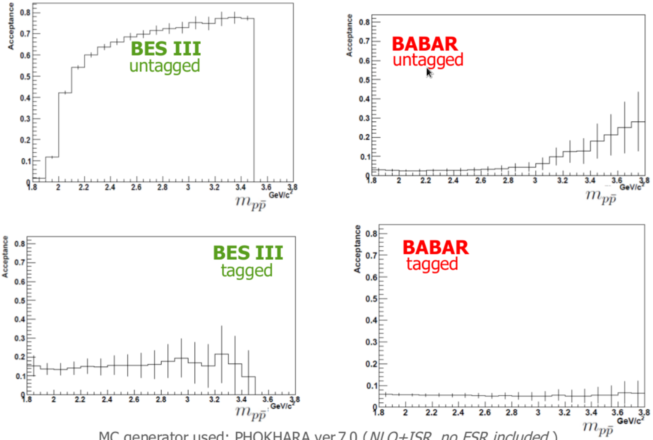
# ISR Geometry




$E_{\gamma,ISR}$ distribution and $\theta$-distribution for $p\,\overline{p}$ at BESIII, Babar

$$Acc_{untagged} = \frac{\#\text{evts with } \mathbf{p,pbar} \text{ in detector, } \gamma \text{ out}}{\#\text{ evts in } L_{BABAR,BES} \text{ for } \sqrt{s}_{BABAR,BES}}$$

$$Acc_{tagged} = \frac{\#\text{evts with } \mathbf{p,pbar,}\gamma \text{ in detector}}{\#\text{ evts in } L_{BABAR,BES} \text{ for } \sqrt{s}_{BABAR,BES}}$$

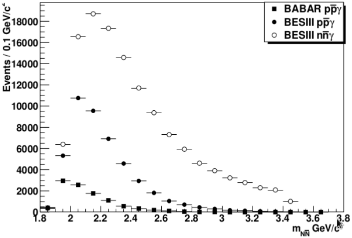



MC generator used: PHOKHARA ver.7.0 (*NLO+ISR, no FSR included*)

Acceptance in tagged and untagged case

- Phokhara standalone (ISR, NLO, no FSR)

| | |
|---|---|
| $\sqrt{s}$ | 3.77 GeV |
| $\sigma_{ISR,NLO}$ | $1.019 \cdot 10^{-3}$ (nb) |
| L | 10 ($fb^{-1}$) |
| $N_{gen}$ | 198153 |
| measurement | tagged+untagged |
| cuts | $21.56° < \theta_{n\bar{n}} < 158.43°$ |
| | $0° < \theta_{\gamma,ISR} < 180°$ |
| $N_{exp}(utag)$ | 128799(28767) |




Expected events for $e^+e^- \rightarrow N\bar{N}\gamma$

# Analysis Strategy

I will start with a tagged analysis, then I will do an untagged one

Particle identification in my channel:

- Select events with no charged tracks originating in the interaction region
- ISR photon: highest energetic cluster at EMC which not an $\overline{n}$
- $\overline{n}$ annihilation signature more identificable than $n$ shower
- Separation of $\overline{n}$ and photon shower (star-topology) in EMC
- Accepted $\overline{n}$ candidate: search for $n$ EMC showers on the opposite side of the detector
- If multiple showers: select most back-to-back with respect to $\overline{n}$ candidate
- ...

# Analysis Strategy

Expectations for Background:

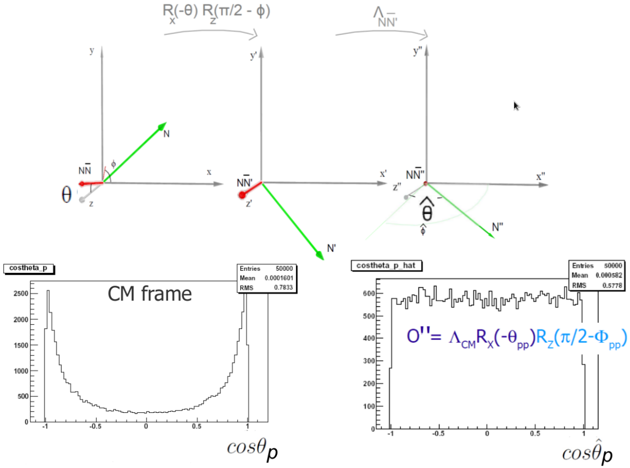
- $e^+e^- \rightarrow n\bar{n}\pi^0$
- $e^+e^- \rightarrow n\bar{n}\gamma_{FSR}$
- $e^+e^- \rightarrow n\bar{n}\gamma\pi^0$
- $e^+e^- \rightarrow J/\Psi\gamma_{ISR},\ J/\Psi \rightarrow n\bar{n}$
- IRS/FSR interference does not contribute to cross section because of different charge parity of the amplitudes corresponding ISR/FSR[1]
- FSR contribution estimated to be $10^{-3}$ of ISR cross section[1]
- ...

[1] Babar analysis document 855 - proton case

# Analysis Strategy

Extraction of the form factors:

- Angular distribution analysis needed
- Transformation into $n\bar{n}$ rest-frame




Transformation into the $n\bar{n}$ rest-frame

# Status and Outlook

- Phokhara standalone installation ✓
  - cross sections to small (Bug?) - recognized yesterday ×
- MC true values simulation (Phokhara standalone) ✓
  - Solving problem with Phokhara standalone (contact Henryk Czyz) ×
- Boss installation in himster, blaster and ihep ✓
- Produced first MC signal samples at himster and ihep (.rtraw, .dst) ✓
- Write first version of analysis routine ✓
  - Many problems with himster → no plots at this time ×
  - Event-selection not ready yet ×
- Backgroung rejection, FFs extraction, systematic studies, ...

Thank You!

| Reaction | $Q^2$ [GeV$^2$] | Observables | Laboratory | Year | Reference |
|---|---|---|---|---|---|
| $e + p \to e + p$ | 4.08-9.59 | $G_{Mp}$ | DESY | 1966 | Albrecht *et al.* [24] |
| " | 0.16-0.85 | $G_{Ep}$, $G_{Mp}$ | SLAC | 1966 | Janssens *et al.* [25] |
| " | 0.69 - 25.03 | $G_{Mp}$ | SLAC | 1968 | Coward *et al.* [26] |
| " | 0.4 - 2 | $G_{Ep}$, $G_{Mp}$ | Bonn | 1971 | Berger *et al.* [27] |
| " | 0.670 -3.01 | $G_{Ep}$, $G_{Mp}$ | DESY | 1973 | Bartel *et al.* [28] |
| | 0.999-25.03 | $G_{Mp}$ | SLAC | 1973 | Kirk *et al.* [29] |
| " | 2.86-31.2 | $G_{Mp}$ | SLAC | 1993 | Sill *et al.* [30] |
| " | 1- 3 | $G_{Ep}$, $G_{Mp}$ | SLAC | 1994 | Walker *et al.* [31] |
| " | 1.75-8.83 | $G_{Mp}$ | SLAC | 1994 | Andivahis *et al.*[13] |
| " | 0.65-5.20 | $G_{Ep}$, $G_{Mp}$ | JLab, E94110 | 2004 | Christy *et al.* [32] |
| " | 0.65-5.20 | $G_{Ep}$, $G_{Mp}$ | JLab, Hall A | 2004 | Qattan *et al.* [34] |
| $\vec{e} + p \to e + \vec{p}$ | 0.49-3.47 | $G_{Ep}/G_{Mp}$ | JLAB, Hall A | 2000 | Jones *et al.* [5] |
| $\vec{e} + p \to e + \vec{p}$ | 3.5-5.5 | $G_{Ep}/G_{Mp}$ | JLAB, Hall A | 2002 | Gayou *et al.* [6] |

TABLE I: Data considered in the present analysis, for proton FFs, in SL region.

| Reaction | $Q^2$ [GeV$^2$] | Observables | Laboratory | Year | Reference |
|---|---|---|---|---|---|
| $ed \to epn$ | 0.04- 1.16 | $G_{En}$ | SLAC | 1965 | Hughes *et al.* [35] |
| $ed \to epn$ | 0.19,0.39,0.56 | $G_{Mn}$ | New York | 1966 | Stein *et al.* [36] |
| $ed \to epn$ | 0.39-0.565 | $G_{Mn}$, $G_{En}$ | DESY | 1969 | Bartel *et al.* [37] |
| $ed \to epn$ | 0.28-1.8 | $G_{Mn}$ | Harvard | 1973 | Hanson *et al.* [38] |
| $ed \to epn$ | 2.5-10 | $G_{Mn}$ | SLAC | 1982 | Rock *et al.* [39] |
| $^3\vec{H}e$ | 0.16 | $G_{En}$ | MIT | 1991 | Jones-Woodward *et al.* [40] |
| $ed \to epn$ | 0.435-1.36 | $G_{Mn}$ | New York | 1964 | Akerlof *et al.* [41] |
| $\vec{D}(\vec{e}, e'\vec{n})p$ | .0.255 | $G_{En}$ | MIT | 1994 | Eden *et al.* [42] |
| $D(e, e'n)$,$D(e, e'p)$ | 0.125-0.605 | $G_{En}$ | Bonn | 1995 | Bruins *et al.* [44] |
| $D(e, e'n)$, $D(e, e'p)$ | 0.235-0.784 | $G_{En}$ | MAMI | 1998 | H. Anklin *et al.* [45] |
| $D(\vec{e}, e'\vec{n})p$ | 0.15 | $G_{En}$ | MAMI | 1999 | Herberg *et al.* [47] |
| $D(\vec{e}, e'\vec{n})p$ | 0.34 | $G_{En}$ | MAMI | 1999 | Ostrick *et al.* [48] |
| $\vec{D}(\vec{e}, e'n)p$ | 0.21 | $G_{En}$ | NIKHEF | 1999 | Passchier *et al.* [49] |
| $^3\vec{H}e(\vec{e}, e'n)pp$ | 0.67 | $G_{En}$ | MAMI | 2003 | Bermuth *et al.* [50] |
| $^3\vec{H}e(\vec{e}, e')$ | 0.1-0.4 | $G_{En}, G_{Mn}$, | JLab | 2000 | Golak *et al.*[52] |
| $\vec{D}(\vec{e}, e'n)p$ | 0.495 | $G_{En}$ | JLab | 2001 | Zhu *et al.* [53] |
| $\vec{D}(\vec{e}, e'n)p$ | 0.5, 1 | $G_{En}$ | JLab | 2004 | Warren *et al.* [7] |
| $D(\vec{e}, e'\vec{n})p$ | 0.3-0.8 | $G_{En}$ | MAMI | 2004 | Glazier *et al.* [9] |
| $D(\vec{e}, e'\vec{n})p$ | 0.5-1.5 | $G_{En}$ | JLab | 1999 | Madey *et al.* [8] |

TABLE II: Data considered in the present analysis, for neutron FFs, in SL region.

| Reaction | $q^2$ [GeV$^2$] | Laboratory | Year | Reference |
|---|---|---|---|---|
| $e^+e^- \to p\bar{p}$ | 4.3 | ADONE, Frascati | 1973 | Castellano *et al.* [54] |
| $p\bar{p} \to e^+e^-$ | 3.52 | CERN | 1977 | Bassompierre *et al.* [55] |
| $p\bar{p} \to e^+e^-$ | 3.61 | CERN | 1983 | Bassompierre *et al.* [56] |
| $e^+e^- \to p\bar{p}$ | 3.75-4.56 | Orsay,DCI | 1979 | Delcourt *et al.* [57] |
| $e^+e^- \to p\bar{p}$ | 4.0-5.0 | Orsay, DCI | 1983 | Bisello *et al.* [11] |
| $p\bar{p} \to e^+e^-$ | 8.9-13.0 | FERMILAB, E760 | 1993 | Armstrong *et al.* [58] |
| $p\bar{p} \to e^+e^-$ | 3.52-4.18 | CERN, LEAR | 1994 | Bardin *et al.* [12] |
| $e^+e^- \to p\bar{p}$ | 3.69-5.95 | ADONE, FENICE | 1994 | Antonelli *et al.* [59] |
| $p\bar{p} \to e^+e^-$ | 8.84 - 18.40 | FERMILAB, E835 | 1999 | Ambrogiani *et al.* [60] |
| $p\bar{p} \to e^+e^-$ | 11.63- 18.22 | FERMILAB, E835 | 2003 | Andreotti *et al.* [2] |
| $e^+e^- \to n\bar{n}$ | 3.61- 5.95 | ADONE, FENICE | 1998 | Antonelli *et al.* [45] |

TABLE III: Data considered in the present analysis for TL region.

# Model from Lomon

$$F_1^v(Q^2) = \frac{N}{2}\left[\frac{1.0317+0.0875(1+Q^2/0.3176)^{-2}}{(1+Q^2/0.5496)}+\frac{g_{\rho'}}{f_{\rho'}}\frac{m_{\rho'}^2}{m_{\rho'}^2+Q^2}\right]F_1^\rho(Q^2)+ \left(1-1.1192\frac{N}{2}-\frac{g_{\rho'}}{f_{\rho'}}\right)F_1^D(Q^2),$$

$$F_2^v(Q^2) = \frac{N}{2}\left[\frac{5.7824+0.3907(1+Q^2/0.1422)^{-1}}{(1+Q^2/0.5362)}+\kappa_{\rho'}\frac{g_{\rho'}}{f_{\rho'}}\frac{m_{\rho'}^2}{m_{\rho'}^2+Q^2}\right]F_2^\rho(Q^2)+ \left(\kappa_\nu-6.1731\frac{N}{2}-\kappa_{\rho'}\frac{g_{\rho'}}{f_{\rho'}}\right)F_2^D(Q^2),$$

$$F_1^s(Q^2) = \left(\frac{g_\omega}{f_\omega}\frac{m_\omega^2}{m_\omega^2+Q^2}+\frac{g_{\omega'}}{f_{\omega'}}\frac{m_{\omega'}^2}{m_{\omega'}^2+Q^2}\right)F_1^\omega(Q^2)+ \frac{g_\phi}{f_\phi}\frac{m_\phi^2}{m_\phi^2+Q^2}F_1^\phi(Q^2)+\left(1-\frac{g_\omega}{f_\omega}-\frac{g_{\omega'}}{f_{\omega'}}\right)F_1^D(Q^2),$$

$$F_2^s(Q^2) = \left(\kappa_\omega\frac{g_\omega}{f_\omega}\frac{m_\omega^2}{m_\omega^2+Q^2}+\kappa_{\omega'}\frac{g_{\omega'}}{f_{\omega'}}\frac{m_{\omega'}^2}{m_{\omega'}^2+Q^2}\right)F_2^\omega(Q^2)+\kappa_\phi\frac{g_\phi}{f_\phi}\frac{m_\phi^2}{m_\phi^2+Q^2}F_2^\phi(Q^2)+ \left(\kappa_s-\kappa_\omega\frac{g_\omega}{f_\omega}-\kappa_{\omega'}\frac{g_{\omega'}}{f_{\omega'}}-\kappa_\phi\frac{g_\phi}{f_\phi}\right)F_2^D(Q^2),$$

with

$$F_1^{\alpha,D}(Q^2) = \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2+\widetilde{Q}^2}\frac{\Lambda_2^2}{\Lambda_2^2+\widetilde{Q}^2},\ \alpha=\rho,\ \omega\ and\ \Lambda_{1,D}\equiv\Lambda_1\ \text{for} F_i^\alpha,\ \Lambda_{1,D}\equiv\Lambda_D\ for\ F_i^D$$

$$F_2^{\alpha,D}(Q^2) = \frac{\Lambda_{1,D}^2}{\Lambda_{1,D}^2+\widetilde{Q}^2}\left(\frac{\Lambda_2^2}{\Lambda_2^2+\widetilde{Q}^2}\right)^2,\ F_1^\phi(Q^2)=F_1^\alpha\left(\frac{Q^2}{\Lambda_1^2+Q^2}\right)^{1.5},$$

$$F_2^\phi(Q^2) = F_2^\alpha\left(\frac{\Lambda_1^2}{\mu_\phi^2}\frac{Q^2+\mu_\phi^2}{\Lambda_1^2+Q^2}\right)^{1.5},\ \widetilde{Q}^2=Q^2\frac{ln[(\Lambda_D^2+Q^2)/\Lambda_{QCD}^2]}{ln(\Lambda_D^2/\Lambda_{QCD}^2)}.$$

# Model from Iachello, Jackson, Lande

$$F_1^s(Q^2) = \frac{g(Q^2)}{2}\left[(1-\beta_\omega-\beta_\phi)+\beta_\omega\frac{\mu_\omega^2}{\mu_\omega^2+Q^2}+\beta_\phi\frac{\mu_\phi^2}{\mu_\phi^2+Q^2}\right],$$

$$F_1^v(Q^2) = \frac{g(Q^2)}{2}\left[(1-\beta_\rho)+\beta_\rho\frac{\mu_\rho^2+8\Gamma_\rho\mu_\pi/\pi}{(\mu_\rho^2+Q^2)+(4\mu_\pi^2+Q^2)\Gamma_\rho\alpha(Q^2)/\mu_\pi}\right],$$

$$F_2^s(Q^2) = \frac{g(Q^2)}{2}\left[(\mu_p+\mu_n-1-\alpha_\phi)\frac{\mu_\omega^2}{\mu_\omega^2+Q^2}+\alpha_\phi\frac{\mu_\phi^2}{\mu_\phi^2+Q^2}\right],$$

$$F_2^v(Q^2) = \frac{g(Q^2)}{2}\left[(\mu_p-\mu_n-1)\frac{\mu_\rho^2+8\Gamma_\rho\mu_\pi/\pi}{(\mu_\rho^2+Q^2)+(4\mu_\pi^2+Q^2)\Gamma_\rho\alpha(Q^2)/\mu_\pi}\right],$$

with $g(Q^2) = \frac{1}{(1+\gamma e^{i\theta}Q^2)^2}$ and $\alpha(Q^2) = \frac{2}{\pi}\sqrt{\frac{Q^2+4\mu_\pi^2}{Q^2}}ln\left[\frac{\sqrt{(Q^2+4\mu_\pi^2)}+\sqrt{Q^2}}{2\mu_\pi}\right]$, with the standard values of the masses $m = 0.939$ GeV, $\mu_\rho = 0.77$ GeV, $\mu_\omega = 0.78$ GeV, $\mu_\phi = 1.02$ GeV, $\mu_\pi = 0.139$ GeV and the $\rho$ width $\Gamma_\rho = 0.112$ GeV.