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Cross Section calculation

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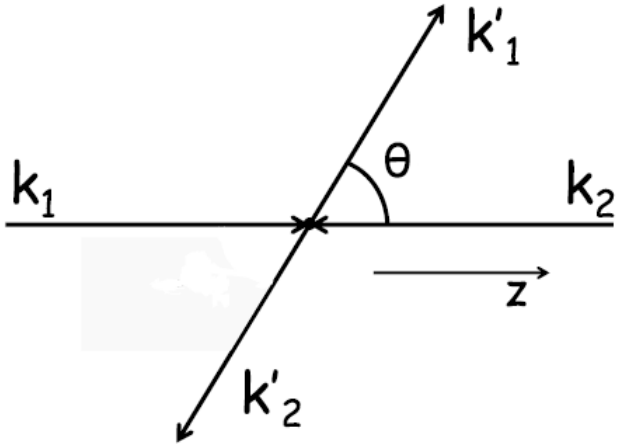
OLYMPUS

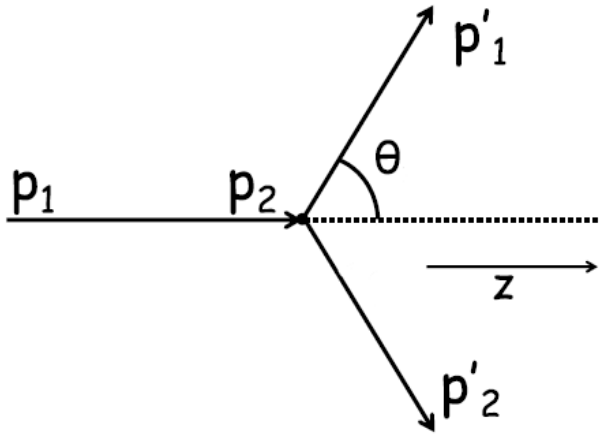
Symmetric Møller/Bhabha Luminosity Monitor

Lab. frame



$$k_1 = (E_{CM}, 0, 0, \sqrt{E_{CM}^2 - m^2}),$$
$$k_2 = (E_{CM}, 0, 0, -\sqrt{E_{CM}^2 - m^2}),$$
$$k'_1 = (E_{CM}, \sqrt{E_{CM}^2 - m^2}(sin\theta.cos\phi, sin\theta.sin\phi, cos\theta),$$

CM frame



$$p_1 = (E, 0, 0, \sqrt{E^2 - m^2}),$$
$$p_2 = (m, 0, 0, 0),$$
$$p'_1 = (E', \sqrt{E'^2 - m^2}(sin\theta.cos\phi, sin\theta.sin\phi, cos\theta),$$

Symmetric Møller/Bhabha Luminosity Monitor

The elastic scattering process can be written using the conservation of the momentum

$$E'(E,\theta) = m\left[\frac{E+m+(E-m)cos^2\theta}{E+m-(E-m)cos^2\theta}\right]$$

Symmetric Møller/Bhabha

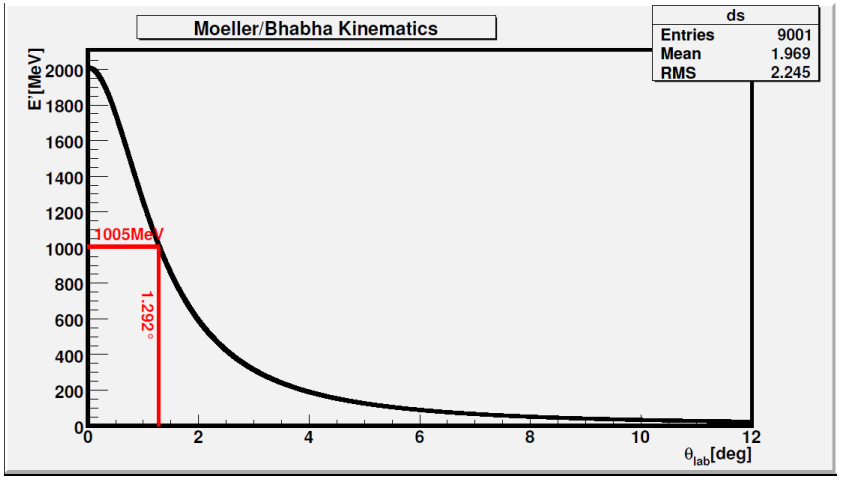
E = 2010 MeV, E' = 1005 MeV
Θ = 1.29199°, m = 0.511 MeV

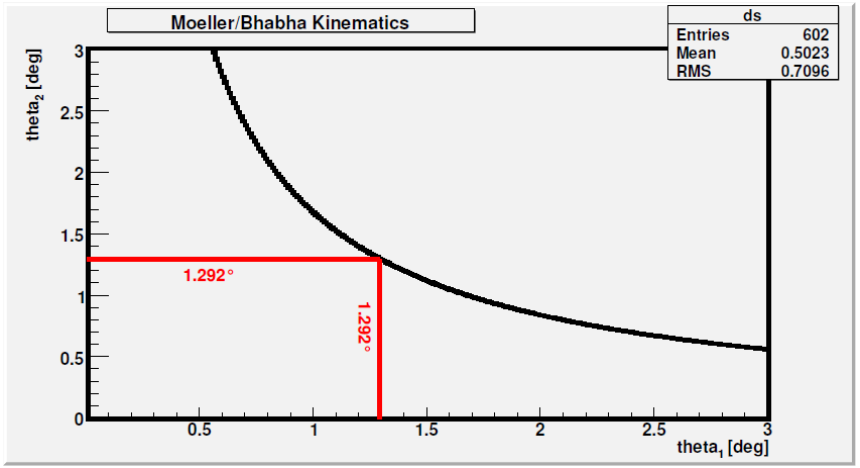
$$E >> m$$

$$E'(E,\theta) \approx \frac{E}{1+\frac{E}{m}(1-cos\theta)}$$

the conservation of energy

$$cos\theta_2 = 1 - \frac{1}{\frac{E^2}{m^2}(1-cos\theta_1)};$$





Cross Section Calculation

$$d\Omega_{lab} = 2\pi sin\theta d\theta_{lab},$$

Lorentz invariant Mandelstam variable
t, u and s

$$\begin{aligned} t &= (p_1 - p_1')^2 = 4m^2 - s - u, \\ &= 4m^2 - 2m(E+m) + 2m(E'-m), \\ &= -2m(E-E') \\ u &= (p_1 - p_2')^2 = -2m(E'-m), \\ &\approx -2mE', \quad for\ E' \gg m \\ s &= (p_1 + p_2)^2 = 2m(E+m), \\ &\approx 2mE, \quad for\ E \gg m \end{aligned}$$

$$\frac{d\sigma}{d\Omega_{lab}} = \frac{d\theta_{lab}}{d\Omega_{lab}} \frac{d\sigma}{d\theta_{lab}},$$

$$\omega = \frac{t}{t+u}$$

can be written as a function of a variable ω

$$\frac{d\sigma}{d\theta_{lab}} = \frac{d\sigma}{d\omega} \frac{d\omega}{d\theta_{lab}}$$

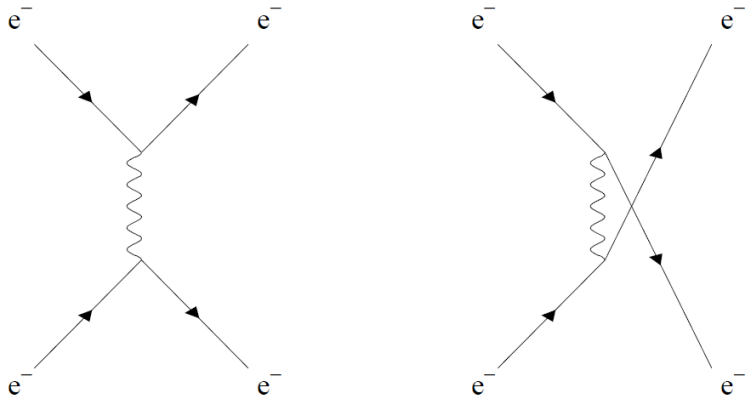
$$\frac{d\omega}{d\theta_{lab}} = \frac{\frac{E}{m} sin\theta_{lab}}{\left[1 + \frac{E}{m}(1 - cos\theta_{lab})\right]^2}$$

$$\frac{d\omega}{d\theta_{lab}} = 22.17251651$$

Symmetric Møller/Bhabha

E = 2010 MeV, E' = 1005 MeV
Θ = 1.29199°, m = 0.511 MeV

Møller Cross Section



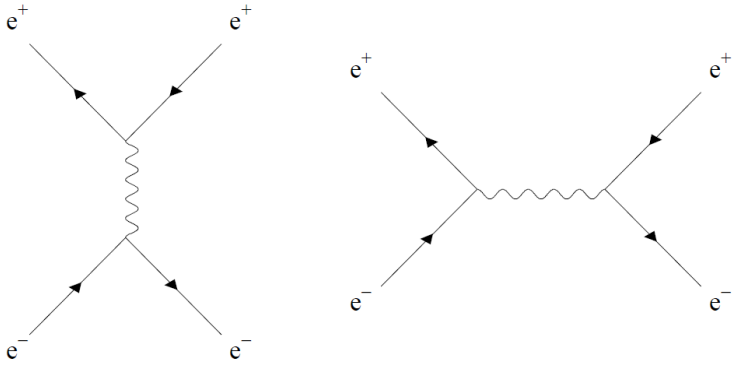
Tree -level graphs for electron-electron scattering

the statistical factor $\frac{1}{2}$ for not distinguishable particles

$$\left.\frac{d\sigma}{d\omega}\right|_{M\text{øller}} = \frac{2\pi\alpha^2}{s}\left(\frac{u^2+s^2}{t^2}+\frac{s^2+t^2}{u^2}+\frac{2s^2}{tu}\right)\cdot\frac{1}{2},$$

$$\left.\frac{d\sigma}{d\Omega_{lab}}\right|_{M\phi ller} = 89{,}38\ \ mbarn/sr$$

Bhabha Cross Section

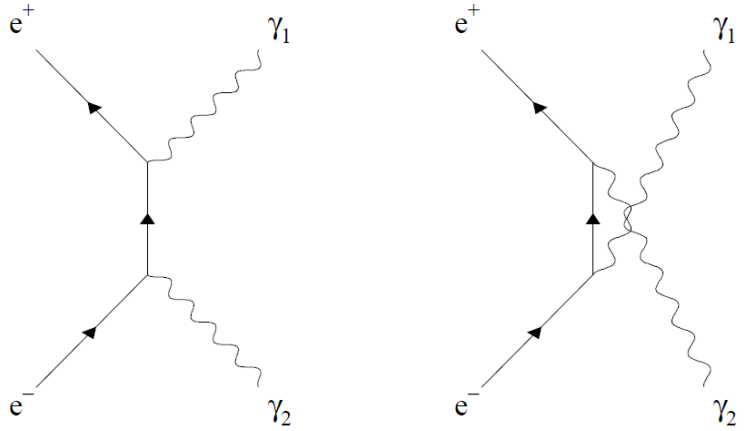


Tree -level graphs for
electron-electron scattering

$$\left.\frac{d\sigma}{d\omega}\right|_{Bhabha} = \frac{2\pi\alpha^2}{s}\left(\frac{u^2+s^2}{t^2}+\frac{u^2+t^2}{s^2}+\frac{2u^2}{ts}\right),$$

$$\left.\frac{d\sigma}{d\Omega_{lab}}\right|_{Bhabha} = 44{,}69\ \ mbarn/sr$$

Annihilation Cross Section



Tree -level graphs for electron-electron scattering

the statistical factor $\frac{1}{2}$ for not distinguishable particles

$$\left.\frac{d\sigma}{d\omega}\right|_{Annihilation} = \frac{2\pi\alpha^2}{s}\left(\frac{u}{t}+\frac{t}{u}\right)\cdot\frac{1}{2},$$

$$\left.\frac{d\sigma}{d\Omega_{lab}}\right|_{Annihilation} = 9{,}93\ \ mbarn/sr$$

Rate

Symmetric Møller/Bhabha Luminosity Monitor

$$\frac{d\sigma}{d\Omega} = \frac{1}{L}\frac{d^2N}{d\Omega_{\text{det}}dt}$$

$$d\Omega_{\text{det}} = \frac{A_{\text{det}}}{d^2}$$

Area of the detector $A_{\text{det}} = \pi\, r^2;\ r^2 = 10\ mm$

Detector distance from the target $d^2 = 3000\ mm$

Luminosity of $L = 2.10^{33} cm^{-2} s^{-1}$

$$N = L\frac{d\sigma}{d\Omega}d\Omega_{\text{det}}$$

Møller ➡ **6,212 KHz**

Single arm rate

Bhabha & Annihilation ➡ **3,119 KHz**

Bhabha & Annihilation ➡ **0,693 KHz**

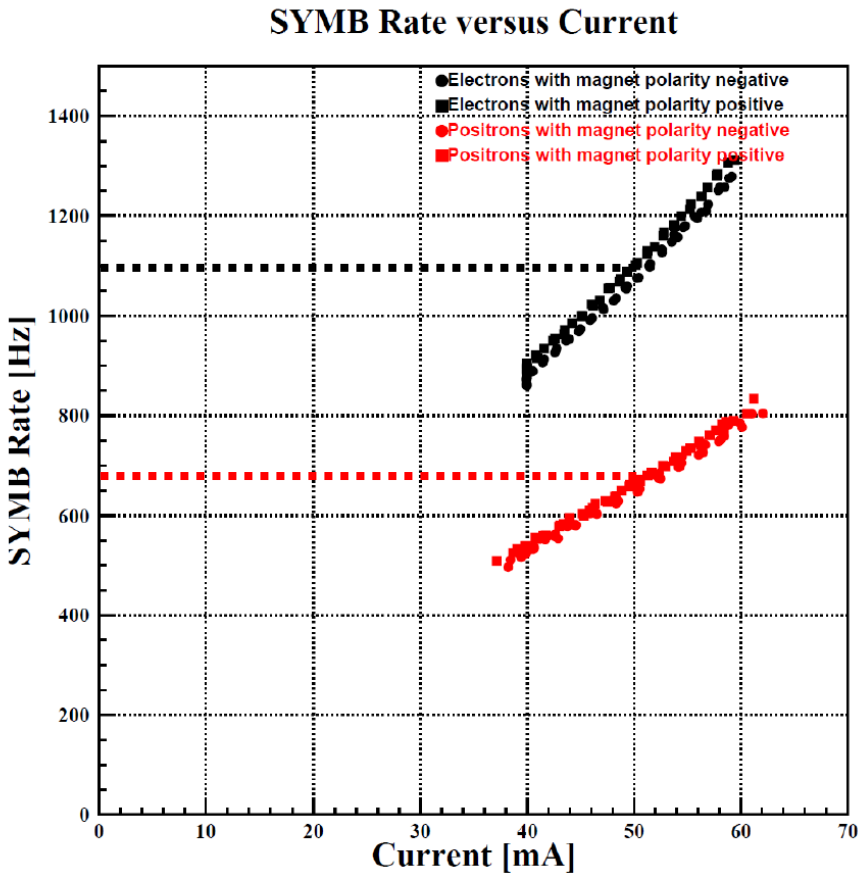
Symmetric Møller/Bhabha Luminosity Monitor

Coincidence rate about 1.5 times less due to solid angle

Møller coincidence rate ~ 4.141 KHz

Bhabha & Annihilation coincidence rate ~ 2.541 KHz

⇨ ~ 1,629



~ 1.100 KHz

~ 0.675 KHz

⇨ ~ 1,629

4.141/1.100 = 3.764545

2.541/0.675 = 3.764444

Symmetric Møller/Bhabha Luminosity Monitor

Symmetric Møller/Bhabha

E = 2000 MeV, E' = 1000 MeV
Θ = 1.295°, m = 0.511 MeV