Normalization of DPM Events v2.0

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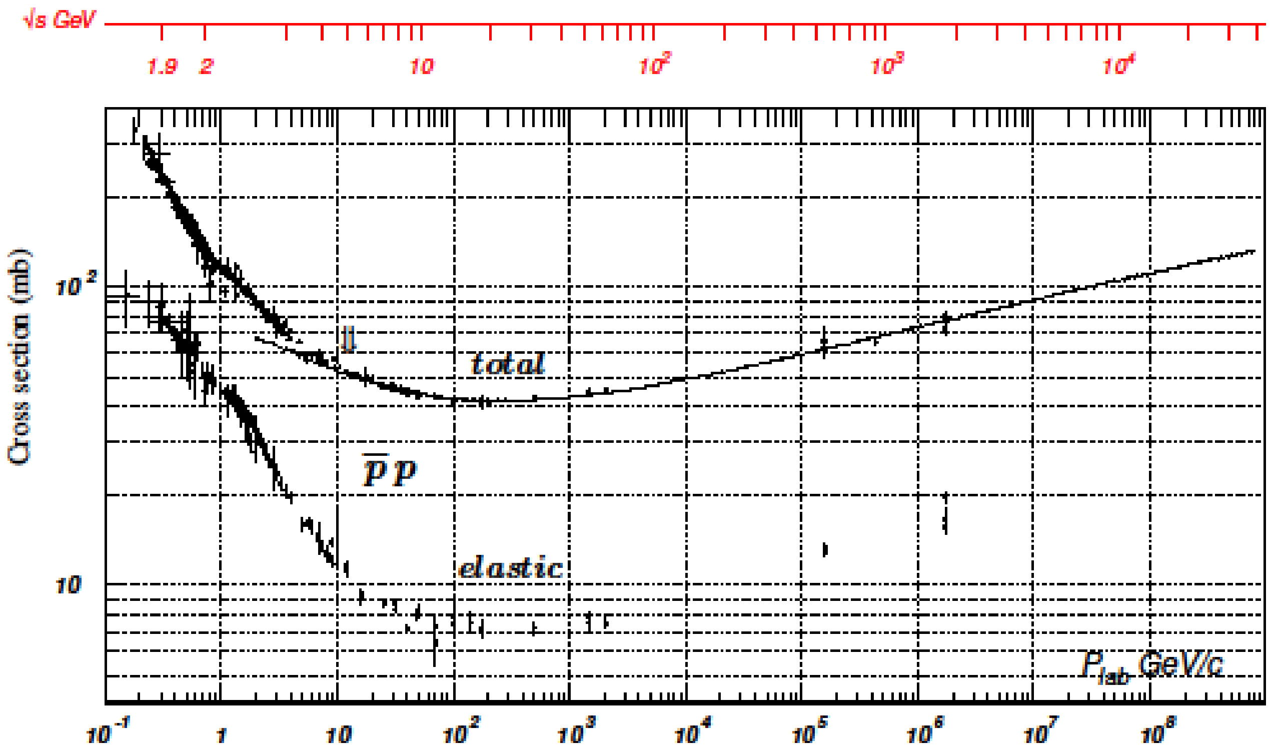
Total Cross Section

The total pbar p cross section can be written as (at a given CM energy $\sqrt{s}$):

$$\sigma_{\text{tot}}^{\text{had}} = \sigma_{\text{inel}} + \sigma_{\text{el}}^{\text{had}}$$

where $\sigma_{\text{tot}}^{\text{had}}$ and $\sigma_{\text{el}}^{\text{had}}$ refer to the hadronic cross sections, i.e. due to strong interaction only!



[K. Nakamura et al. (PDG), J. Phys.G 37, 075021 (2010)]

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Total Cross Section

Determination of total cross section:

- measure total interaction rate

$$\sigma_{\rm tot} = \frac{1}{\mathcal{L}} R_{\rm tot}$$

- measure differential elastic (hadronic) cross section

$$\frac{d\sigma}{dt} = \frac{1}{\mathcal{L}} \frac{dR_{\rm el}}{dt}$$

and use optical theorem

$$\sigma_{\rm tot} = \frac{4\pi}{p} {\rm Im} f_{\rm el}(k, 0)$$

Elastic Scattering

For the **reaction 1+2→3+4**, the 4-momentum transfer can be written as

$$t = (p_1 - p_3)^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\boldsymbol{p}_1||\boldsymbol{p}_3|\cos\theta$$

where θ is the scattering angle (holds for any reference system)

In the CM frame: $0 \le \theta \le \pi$

$$t_0 \equiv t(\theta = 0) = (E_1 - E_3)^2 - (|\boldsymbol{p}_1| - |\boldsymbol{p}_3|)^2$$

$$t_1 \equiv t(\theta = \pi) = (E_1 - E_3)^2 - (|\boldsymbol{p}_1| + |\boldsymbol{p}_3|)^2$$

For **pbar p elastic scattering** we have (in the CM frame):

$$m_i \equiv M,\ \boldsymbol{p}_1 = -\boldsymbol{p}_2,\ \boldsymbol{p}_3 = -\boldsymbol{p}_4,\ |\boldsymbol{p}_i| \equiv P,\ E_i \equiv E$$

$$\Rightarrow t = -2P^2(1 - \cos\theta_{CM})$$

$$\Rightarrow -4P^2 \le t \le 0$$

Elastic Scattering

With $$\frac{d\sigma_{\mathrm{el}}}{d\Omega} = |f_{\mathrm{el}}|^2 = (1+\rho^2)\mathrm{Im}^2\, f_{\mathrm{el}} \quad ; \quad \rho = \frac{\mathrm{Re}\, f_{\mathrm{el}}}{\mathrm{Im}\, f_{\mathrm{el}}}$$

and $$\frac{d\sigma_{\mathrm{el}}}{dt} = \frac{d\sigma_{\mathrm{el}}}{d\Omega}\frac{d\Omega}{dt} = \frac{d\sigma_{\mathrm{el}}}{d\Omega}\frac{\pi}{p^2}$$

we get $$\frac{d\sigma_{\mathrm{el}}}{dt} = \frac{\pi}{p^2}(1+\rho^2)\,\mathrm{Im}^2 f_{\mathrm{el}}$$

and therefore $$\sigma_{\mathrm{tot}} = \frac{4\pi}{p}\,\mathrm{Im} f_{\mathrm{el}}(0) = \sqrt{\frac{16\pi}{1+\rho^2}\left.\frac{d\sigma_{\mathrm{el}}}{dt}\right|_{t=0}}$$

Elastic Scattering

Include also Coulomb interaction in the amplitude:

$$\frac{d\sigma_{\text{el}}}{dt} = \frac{d\sigma_{\text{el}}^{\text{C}}}{dt} + \frac{d\sigma_{\text{el}}^{\text{int}}}{dt} + \frac{d\sigma_{\text{el}}^{\text{had}}}{dt}$$

$$= \pi \left(\frac{2\alpha}{t}\right)^2 G^4(t) - 4\pi \frac{\alpha}{|t|}(\rho + \alpha\phi) G^2(t) \left(\frac{1}{1+\rho^2} \frac{d\sigma_{\text{el}}^{\text{had}}}{dt}\right)^{1/2} + \frac{d\sigma_{\text{el}}^{\text{had}}}{dt}$$

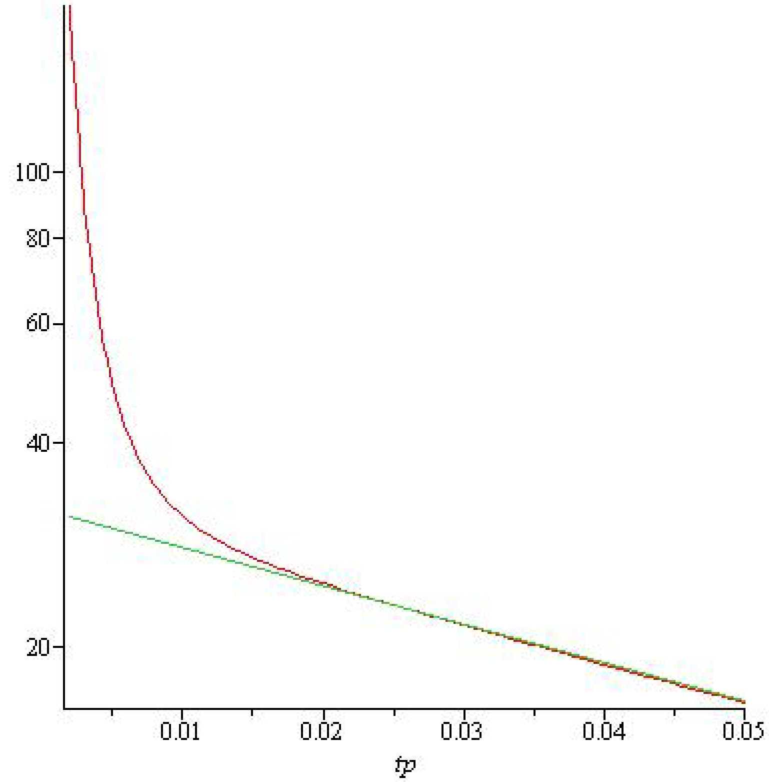
Hadronic elastic scattering:

$$\frac{d\sigma_{\text{el}}^{\text{had}}}{dt} = \left.\frac{d\sigma_{\text{el}}^{\text{had}}}{dt}\right|_{t=0} e^{-b|t|}$$

Fit above function to measured $\frac{d\sigma_{\text{el}}}{dt}$ with b, $\left.\frac{d\sigma_{\text{el}}^{\text{had}}}{dt}\right|_{\text{t=0}}$ as free parameters

$$\Rightarrow \sigma_{\text{tot}}^{\text{had}} = \sqrt{\frac{16\pi}{1+\rho^2} \left.\frac{d\sigma_{\text{el}}^{\text{had}}}{dt}\right|_{\text{t=0}}}$$

Elastic Scattering



Coulomb Scattering

The Coulomb differential elastic cross section diverges for 4-momentum transfer $t \to 0$

The cross section observed in any experiment is finite because of finite resolution / acceptance at very small angles.

This is why a cut-off angle $\theta_{\min}$, corresponding to a minimum value $|t|_{\min}$ of $-t$, is introduced in DPM.

This cut-off is applied to all elastic events, not only Coulomb events.

Normalization

The rate of inelastic interactions is fully determined by the cross section

$$R_{\text{inel}} = \sigma_{\text{inel}} \cdot \mathcal{L}$$

Switching on elastic scattering with a given cut-off angle $\theta_{\min}$, the rate is

$$R_{\text{DPM}} = \sigma_{\text{DPM}} \cdot \mathcal{L}$$ with $$\sigma_{\text{DPM}} = \sigma_{\text{inel}} + \sigma_{\text{el}}\left(|t| \geq |t|_{\min}\right)$$

$$\sigma_{\text{el}}\left(|t| \geq |t|_{\min}\right) = \int_{t_1}^{-|t|_{\min}} \frac{\mathrm{d}\sigma_{\text{el}}}{\mathrm{d}t}\,\mathrm{d}t$$

DPM reproduces correctly the ratio $\dfrac{\sigma_{\text{el}}\left(|t| \geq |t|_{\min}\right)}{\sigma_{\text{inel}}}$ for given value of |t|

⇨ Normalization can be based entirely on σ_{inel}

Normalization

In DPM $\sigma_{\mathrm{el}}\left(|t| \geq |t|_{\min}\right) = \sigma_{\mathrm{el}}^{\mathrm{C}}\left(|t| \geq |t|_{\min}\right) + \sigma_{\mathrm{el}}^{\mathrm{had}}\left(|t| \geq |t|_{\min}\right) + \sigma_{\mathrm{el}}^{\mathrm{inter}}\left(|t| \geq |t|_{\min}\right)$

describes quite well the experimental data up to |t|~2 GeV^2

Therefore, the following situation should be avoided:

- $\theta_{\min}$ is chosen so small (i.e. $|t|_{\min}$ so small) that the elastic cross section is completely saturated by the (diverging) Coulomb part of the cross section
- the ratio of elastic to inelastic events would still be correct, since it is determined by data, but no hadronic elastic events would be present in the data sample (which are important for the CT studies!)

Determination of $|t|_{\min}$

Choose a cut-off angle $\theta_0 < \theta_{\min}$, but still large enough, such that

- the Coulomb part does not saturate the elastic cross section
- there are still hadronic elastic events in the data sample

⇨ approximately in the interference region of the elastic cross section, i.e. $t \sim 10^{-3}$ (GeV/c)^2

Determination of $|t|_{min}$

In order to determine the value of $|t|_{min}$ to be used for the CT studies, there are two options:

- demand that the proton track is able to leave the beam pipe
- demand that the proton track is able to leave the MVD

Many low-energy tracks would not be observed in the CT for the first option, but they would produce a high occupancy in the MVD. They may be important if one wants to associate events in the CT to hits in the MVD.

Therefore, the first option is more realistic, although it corresponds to a smaller cut-off angle