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χ^2

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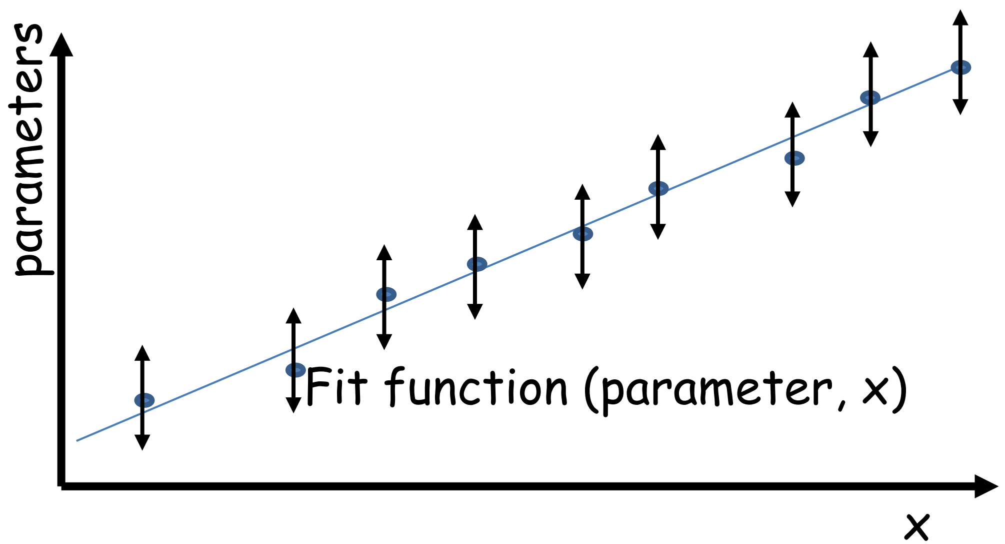
The χ^2 distribution

The χ^2 variable is defined as:

$$\chi^2 = \sum_{i=1}^{f} \frac{x_i^2}{\sigma_i^2}$$

x_i^2 → independent, normally distributed variables with zero mean

σ_i^2 → variance



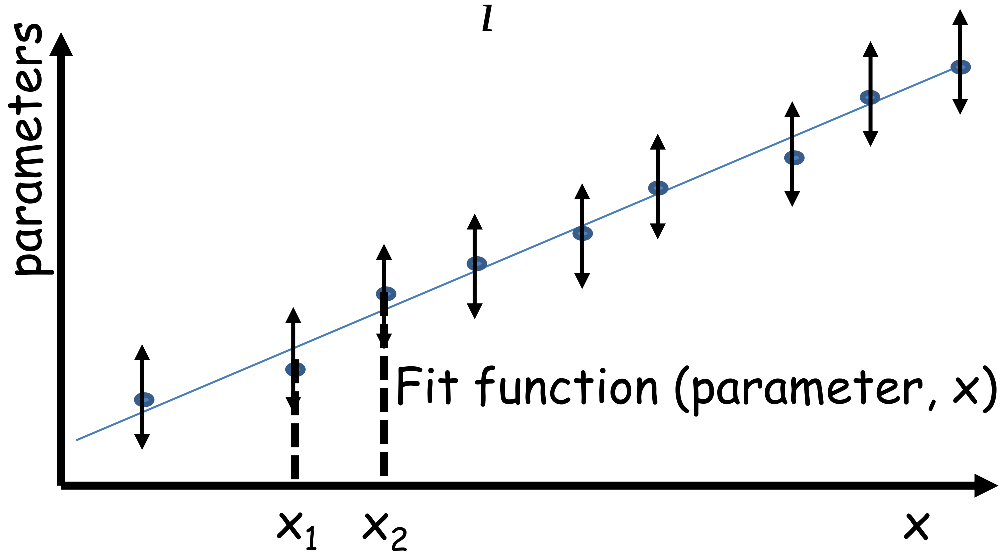
$$\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(Rv-Mv)^2}{2\sigma^2}}$$

Rv :Real Value
Mv :Measured Value

Ff : Fit function

$$-\ln\left(\frac{1}{2\pi\sigma_{x1}} e^{-\frac{(Ff(par,x1)-Mv1)^2}{2\sigma^2{}_{x1}}} \cdot \frac{1}{2\pi\sigma_{x2}} e^{-\frac{(Ff(par,x2)-Mv2)^2}{2\sigma^2{}_{x2}}} \right)^2$$

$$\chi^2 = \sum_i \frac{(Ff_i - Mv_i)^2}{\sigma^2{}_i}$$



The variate χ^2 is defined as the sum

$$\chi^2 = \sum_{i=1}^{f} \frac{x_i^2}{\sigma_i^2} ,$$

where x_i are independent, normally distributed variates with zero mean and variance σ_i^2.

We have already come across the simplest case with $f = 1$ in Sect. 3.4.1: The transformation of a normally distributed variate x with expected value zero to $u = x^2/s^2$, where s^2 is the variance, yields

$$g_1(u) = \frac{1}{\sqrt{2\pi u}} \mathrm{e}^{-u/2} \quad (f = 1) .$$

(We have replaced the variable χ^2 by $u = \chi^2$ to simplify the writing.) Mean value and variance of this distribution are $E(u) = 1$ and $\mathrm{var}(u) = 2$.

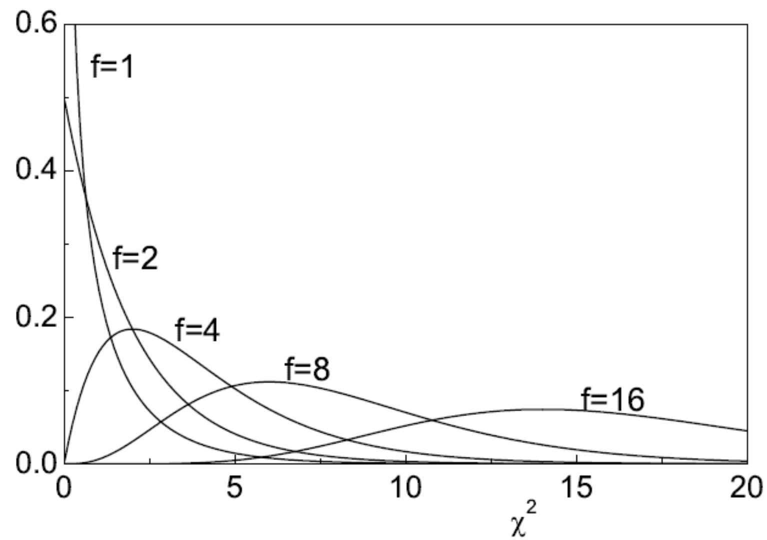


Fig. 3.20. χ^2distribution for different degrees of freedom.

http://www-library.desy.de/preparch/books/vstatmp_engl.pdf